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An analysis of eleemosynary behavior among individuals

by

Michael David Packard

A Dissertation Submitted to the

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Iowa State University
Ames, Iowa

1977

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CHAPTER I. INTRODUCTION

It is estimated that in 1975 living individuals contributed $21.4 billion to philanthropic causes [1, p. 6]. In the same period disposable personal income was $1076.8 billion so that giving as a percentage of disposable personal income was 1.99 [1, p. 6]. This estimate of individual contributions probably understates the true level of giving or voluntary transfer since it is based upon IRS tax data which takes into account only itemized deductions due to contributions and an estimate of the contributions of nonitemizers. It does not take into account transfers to individuals or transfers to organizations that do not hold a tax deductible status, nor does it take into account the value of the time that individuals donate to various causes. It is estimated that Americans gave more than five-and-a-half billion hours of volunteer time to various charitable causes and organizations during 1975. The value of this time is estimated to be worth more than $25 billion [1, p. 8]. Americans are a charitable people, both in terms of the money and goods which they annually donate to various causes and in terms of the time which they voluntarily give to these same and other causes.

That portion of economic theory known as the theory of the consumer usually assumes that the consumer is selfish in that his utility function depends only upon the goods and services that he himself consumes. It is assumed that the consumer always wants more of a good or service rather than less of it. This implies that the consumer is never sated with what he has and that he will never give away the goods that he possesses nor his purchasing power (money) unless he receives something in return on which he places a value equal to or greater than the value of the item which he gave away. If we assume that gifts are one-way transfers, i.e., something is given but nothing is received, then a rational consumer would not give away items dear to him. The fact that consumers apparently voluntarily give away some two percent of their annual income seems to contradict a general assumption about consumer behavior.

This dissertation will concern itself with a study of why consumers voluntarily transfer purchasing power or ownership of goods and services to other individuals, groups, and organizations. The redistribution of resources may be effected in several ways. Perhaps the most common is through a change in relative prices. If an economic system were initially in equilibrium and relative prices changed, then resources would tend to flow into those areas that had experienced relative price increases and out of those areas that had experienced relative price decreases. This redistribution of resources is, of course, brought about by the profit motive.

Another method of redistributing resources is through the tax-transfer mechanism available to governmental units. Governmental units reduce the resource holdings of consumers and producers through taxation and then use these acquired resources either to purchase goods and services or to make transfers among producers and consumers under that governmental unit's jurisdiction. If the governmental unit uses the resources to purchase goods and services, the bundle of commodities that it purchases will, in general, differ from the bundle that the taxed consumers and producers would have purchased. Thus, resources will flow to the producers of the commodity bundle purchased by the governmental unit and away from the producers of the commodity bundle that the taxed consumers and producers would have purchased.

A third method of redistribution is through voluntary transfers by individuals. Resources are transferred from those who make the transfer to those who receive the transfer. We are concerned with the last of these three methods of redistributing resources, with what Musgrave has labeled <u>secondary redistribution</u> and what Boulding has denoted private grants [5, p. 6; 38, p. 991]. Redistribution through the public sector has been labeled <u>primary redistribution</u> by Musgrave and public grants by Boulding [5, p. 6; 38, p. 991].

My interest in the area of voluntary transfers stems from Boulding's work on "grants economics" [5] fortified by the ever increasing volume of literature that concerns itself with Pareto optimal redistribution.[1] Boulding has argued that an economic theory based solely upon the concept of exchange paints an incomplete picture of the state of the world. A more complete picture would be presented if we included a concept of grants within the economic framework. Boulding defines grants as one-way transfers of economic goods which may be motivated by either integrative (loving) relationships and the integrative system or threats and the threat system. We shall generally assume that transfers are motivated by the integrative system, for if the threat system were in force the voluntary nature of transfers would be lost or, at least, impaired.

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[1]For a recent article with an extensive bibliography on Pareto optimal redistribution see Johnston [30].

Boulding lists several areas where a theory of grants could improve economic theory in general. The most obvious of these areas is in the field of philanthropy and, by extension, governmental welfare programs. He states that "Economics has a theory of the firm, as it exists in an exchange economy; it has no theory of a foundation, and no very good theory of a government as an economic organization, partly because of its neglect of the grants concept" [5, p. 5]. Another area that could be improved by a theory of grants is the theory of organizations. Internal transfers of resources are of great importance to the functioning of organizations of all sizes, from the family up to and including the socialist state. Finally he points out that the grants concept could be used in analyzing the socialist controversy and other political controversies. At the heart of the socialist controversy is the question of what proportion of the activities of society should be organized by exchange and what proportion by grants. Our interest lies mostly in the first of these possible uses of the grants concept.

The literature that concerns itself with redistribution from a Pareto efficient viewpoint is a fairly recent development. In the late 1960's and early 1970's several authors independently discovered that the Pareto criteria could be effectively used in the study of redistribution [2; 19; 22; 34; 41; 48]. The Pareto criterion can be stated as follows. If we are given two states of nature, state A and state B, and if society is originally in state A, then a move from state A to state B is a Pareto better move if 1) at least one person feels himself to be better off in state B than in state A <u>and</u> 2) no other person feels himself to be worse off in state B compared with state A. The great weakness of the Pareto criterion is that it can not tell us which state is preferred if in moving from one state to another some people are made worse off and some people are made better off. In order to distinguish a preferred state in this instance it would be necessary to compare utility gains and losses for the entire societal population. If the sum of the utility gains were greater than the sum of the utility losses, then the new state would be the preferred state. This type of analysis implies that we could objectively make interpersonal utility comparisons. In his famous essay, Robbins [42] argued that such interpersonal utility comparisons could not be made objectively. Indeed, his essay marked the end of the "old" welfare economics which had assumed that such interpersonal utility comparisons could be made.

It was long thought that the inability of the Pareto criterion to choose a preferred state when there were some gainers and some losers meant that it was unable to make any distributional judgments whatsoever. This distributional hypothesis was based, however, upon the assumption that the utility functions of all consumers were independent. Once we allow utility functions to become interdependent, then the possibility of Pareto relevant redistribution occurs. Most generally the interdependence takes the form of consumption externalities whereby a subset of the commodity bundle consumed by one individual will affect the utility level of some other individual [2; 13; 41]. Occasionally the interdependence will be a utility externality where the utility level of one individual will enter as an argument in the utility function of another individual [22; 49]. Clearly the most general interdependent utility function will incorporate both the consumption externality and the utility externality [12; 43].

Utility interdependence or interdependence of preferences is not a new concept, but until the recent surge of interest in how it relates to redistribution, this interdependence concerned itself with how arguments in the utility function of one individual might affect the tastes of another individual as they relate to his own (the second individual's) consumption of goods and services [15; 29; 32; 37]. Other than this effect on tastes, interdependence of preferences was seen as a thorny problem by the theorist and was assumed away whenever possible. The chief reason for neglecting interdependence of preferences is that this interdependence can lead to situations where not every competitive equilibrium will be Pareto efficient [2, p. 385; 11, p. 134]. When utility interdependence is present Pareto efficiency may require the price of goods be different for different consumers.

The two person case for redistribution when interdependence of preferences is present is reasonably straightforward, but due to the free rider problem[1] and the pricing problem discussed above the analysis does not extend easily to the many person case. Some economists who have studied this problem recommend that redistribution be made a collectivized good and that this collectivization be achieved through the public or political market [22, p. 543; 38, p. 991; 48, p. 335; 49, p. 629].

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[1]The free rider problem occurs when an individual receives benefits from consuming a good or service but does not pay his share of the cost of providing the good or service.

This extension into the political market raises many problems for the economist, however.

One problem is very familiar to economists and has to do with the controversy over whether taxes should be based on the benefit principle or the ability-to-pay principle. For Pareto efficient redistribution it is necessary that taxes be levied on the basis of the benefit that taxpayers receive from such redistribution.

The benefit principle is based on the fact that individuals receive some marginal benefit from the output of a given amount of public goods (and here we are including redistribution as a public good). If the government could ascertain the marginal benefits accruing to each consumer it would be able to assign and collect taxes on the basis of these true marginal benefits. With perfect knowledge the government could adjust the output of public goods in such a way that the taxes which were collected on the basis of the marginal benefits to taxpayers were just sufficient to cover the costs of that particular bundle of public goods. Thus, in theory, the moving of redistribution from the private sector to the public sector could solve the free rider problem because theoretically each person could be taxed according to the marginal benefits he received from redistribution. In reality taxpayers will not reveal their true preferences and the government has no practical way of divining what their true preferences are. Thus we move from the free rider problem of the private sector to the preference revelation problem of the public sector.

One could argue that even with the preference revelation problem the collectivization of redistribution would be more efficient that leaving it in the private sector since there would be fewer free riders in the public sector and total redistribution would be closer to the optimum. If we assumed taxes were based on the ability-to-pay principle then some individuals would be taxed according to their marginal benefit from redistribution, some would be undertaxed, and some would be overtaxed, but total monies collected for redistribution should increase. Even if the collective redistribution of transfers were made in accordance with taxpayers' private preferences we still could not be certain that collective redistribution would be Pareto better than purely voluntary redistribution. When all interdependencies are benevolent, the voluntary movement from the pre-transfer distribution to the post-transfer distribution must be a Pareto better move even if the Pareto efficient locus[1] is not reached. In the tax-transfer case, the movement from the original, pre-tax distribution to the final, post-transfer distribution is not necessarily a Pareto better move. We can assume that the individuals who are undertaxed and properly taxed will be better off, especially if those who are undertaxed have the opportunity to make nontax transfers. Those who are

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[1]A distribution is on the Pareto efficient locus if it is impossible to redistribute any good without making some person worse off.

overtaxed will not necessarily be better off and might be worse off under the post-transfer distribution than they are under the pre-tax distribution. If they are worse off, then we are back to the major weakness of the Pareto criterion. It can not tell us whether the pre-tax or post-transfer distribution is preferred if some members of society are made better off by the move and some members are made worse off. Moving redistribution from the private sector to the public sector does not guarantee that the Pareto efficient locus will be reached or even that the final collective distribution will be preferred to the final voluntary distribution.

There is an alternative to redistribution as presented above. Instead of redistributing resources through the public sector or by individual action in the private sector, a group of individuals may form within the private sector for the purpose of effecting such redistribution as the members of the group desire. These private sector groups will be called charities. The particular economics of charities have not been widely studied.[1] We will discuss this particular group in Chapter III.

Individuals are driven to join or contribute to charities or to make individual voluntary transfers by many different motives and objectives. We will generally assume that the word "voluntary" means that the individual making a transfer is not coerced into making such transfer and that he does not receive any socially-imposed penalty if he does not make

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[1] Ireland and Johnson [27] is the only reference we have found that devotes itself to charities as we have defined them.

a transfer. Noncoercive social pressure is a gray area that we will discuss below. Some of the motives and objectives that cause individuals to make transfers are listed below.

1. Individuals may have interdependent utility functions where the interdependency consists of consumption externalities, utility externalities, or both. This motivation has been discussed briefly above and will be discussed in more detail in later chapters.

2. Giving appears as an argument in the utility functions of individuals [5, p. 3; 26, p. 72; 27, p. 20; 28, p. 93; 35, p. 4; 48, p. 327]. In this case the act of giving renders utility to the individual. This motive is sometimes called the Kantian motive [27, p. 20; 28, p. 93]. It is not clear that the giver of the transfer would necessarily receive more utility from making a larger transfer as opposed to a smaller one in this case. That is, with this motive there may be a region of diminishing marginal utility from transfers. The point at which the diminishing marginal utility occurs would probably depend upon the perceived importance of the "cause" toward which the transfer was directed. Also, in this case, the initiator of the transfer would probably not have any systematic method for determining the recipient of the transfer. Since the utility is received from the act of giving, the recipient would seem to be of secondary importance.

3. The distribution of income or resources enters as an argument in individual utility functions [5, p. 8; 6, p. 14; 23, p. 65; 47, p. 1; 48]. Presumably the desire for a particular income distribution will motivate the individual in question to make transfers in order to bring

about his desired distribution. The amount the individual can transfer is constrained by his initial income holdings and by the fact that his transfers will be limited to the point at which his loss in utility due to the reduction in his own income is just equal to his gain in utility from achieving a more desired distribution of income.

4. The individual may have religious motives for making transfers [20, p. 13; 28, p. 92; 35, p. 4; 36, p. 15]. Historically, this is probably the most important motivating force behind personal transfers and, although this force is probably not as strong now as in years past, it is estimated that between forty and fifty percent of all charitable contributions in the United States flow through religious organizations [1, p. 7]. Religions influence an individual's tastes by stressing to him the importance of looking after his fellow man. The religious motivation is reinforced by the following motivations which will be explained more fully below: 1) the insurance motive--an individual will give to a religious organization with the hope that the organization would provide for him should his position deteriorate substantially; 2) the long run exchange motive--the individual is willing to forego personal consumption here on earth in order to insure himself a place in heaven, i.e., he is exchanging temporal wealth for spiritual wealth; and 3) the social pressure motive--since the local units of religious organizations are usually organized as small number groups, the problems that arise in large number groups can be avoided. The free rider problem will not be as prevalent since group members can bring social pressure to bear upon those members who attempt to free ride. The fact that the individual has the option of

withdrawing from the religious group but does not do so would indicate that he is in basic agreement with the policies of the organization. Hence, he would be more likely to make voluntary donations to that organization with a minimal amount of social pressure than if the withdrawal option were not present.

5. The insurance motive can also cause the individual to make voluntary transfers. This motive has been presented in several forms: disaster relief [5, p. 4; 14]; revolution avoidance [26, p. 72; 35, p. 168; 36, p. 14; 48, p. 327]; maintenance of allegiances [36, p. 71]; the religious motive [28, p. 92]; and income stability [7, p. 44; 44, p. 166; 48, p. 327] among others. All of these motives have one common goal--to maintain a relatively constant utility level over the life cycle. Individuals will be willing to make transfers when their utility levels are reasonably high and when they have the knowledge or expectation that doing so will make them eligible to receive transfers when their utility falls to low levels and they think that there is a positive probability that their utility will fall to low levels.

6. Many transactions thought to be one-way transfers may not be grants at all, but may be, in reality, intertemporal exchanges [5, p. 2; 36, p. 14]. In this case grants take on the meaning of credit given or credit repaid.

7. Some transfers may be motivated by social pressure [3, p. 166; 24, p. 282; 26, p. 73; 28, p. 94]. If the individual received no utility or negative utility from making the transfer then we assume that he would not make the transfer even in the face of social pressure. This implies

that the potential transferor does not consider a cost to be attached to the social pressure. In reality, the individual will probably attach a cost to social pressure and, in this case, he will make the transfer if the cost of doing so is less than the cost of the social pressure. One might consider threats to be a kind of social pressure, but threats negate the voluntary aspect of transfers and we have stated that the type of transfer we wish to study is the voluntary type.

8. Transfers may be seen as rewards for certain types of behavior on the part of the recipient [5, p. 3; 24, p. 284]. Managers who pass out bonuses to their employees and parents who base transfers to their children in whole or in part upon the children's behavior would fall into this category.

9. Transfers may be given in order to reduce one's tax burden [16; 17; 35, p. 4]. Indeed, it has long been recognized that the progressive tax rate and the right of individuals to deduct some charitable contributions from their tax base combine to make the price of charitable contributions decline as the individual's income increases. Thus high income (wealthy) individuals have an incentive to transfer more money and goods to charitable organizations than low income (poor) individuals, not only because their income constraint is less binding, but also because the price of the contribution is lower for them.

10. Transfers may be given to free the transferor from some nuisance presented by the recipient [46, p. 1265]. Giving a donation to a beggar on the street or to someone who knocks on one's door would be of this type of motivation if the <u>sole</u> reason for the transfer was to get rid of

the recipient or of some nuisance associated with him. In this particular case there is actually a two-way exchange taking place but since the gain to the donor (i.e., nuisance relief) cannot be observed the exchange is misclassified as a one-way transfer. Transfers made due to social pressure might be classified as nuisance relief transfers insofar as social pressure is a nuisance to the potential transferor. Also transfers made because of threat or force applied to the transferor by the recipient would fall into this category if the force or threat were removed when the transfer was made.

11. An individual may make a transfer, not out of any concern for the recipient, but with the knowledge that by making such a transfer he improves his own self-esteem and/or his level of prestige within his community [3, p. 166; 5, p. 3; 25, p. 73; 26, p. 454; 35, p. 4; 36, p. 1; 46, p. 1264]. This is different from the Kantian motive (motive 2) in that it is not so much the act of giving that yields utility to the transferor as it is his knowledge that his transfers will increase his status within the community.

12. The last motive to be mentioned is that individuals will make transfers in order to put the recipients in their debt [3, p. 166; 25, p. 454; 35, p. 4; 36, p. 1]. Very often giving is a means of obtaining social credit whereas receiving a gift signifies accepting a social debt. If the transferor desires to control the recipient then the recipient's acceptance of the gift will indicate that he is willing to accept the conditions put forth by the transferor.

The motivations and objectives presented above certainly do not include all the possible reasons that would cause an individual to make transfers, but we feel that these are some of the major reasons. The many and varied motives and objectives that drive individuals to make voluntary transfers and the complex inter-relationships among these motives and objectives make it extremely difficult to develop a single methodology that will adequately explain voluntary consumer transfers. We feel that the interdependent utility function offers the most reasonable and most promising approach to follow in developing a theory of consumer transfers. This is the approach we will follow.

In the next chapter we will develop a two person model of eleemosynary behavior utilizing interdependent utility functions. We will compare the optimal solutions when only one individual experiences utility interdependence, and when both individuals experience utility interdependence. We will then show how these solutions differ from the solutions obtained when the analysis assumes independent utility functions.

In Chapter III we will attempt to generalize our model to the N-person case. We will study the case where only one individual experiences utility interdependence and some cases where many or all individuals have interdependent utility functions. In many instances it will be to the advantage of potential donors to form groups called charities in order to effect more efficient redistribution. We will study the conditions necessary for the formation of charities and some of the problems that their existence entails. We will also have something more to say about the collectivization of redistribution through the public sector.

The final chapter, Chapter IV, will consist of a summary of results and the conclusions that can be drawn from the analyses presented in Chapters II and III. We will also indicate in Chapter IV what we feel are fruitful directions for further research.

CHAPTER II. THE TWO PERSON CASE

In this chapter we will analyze eleemosynary behavior in a two person world. In order to effectively analyze this type of behavior we must make some changes in the commonly received theory of consumer choice. Specifically we will modify the consumer's utility function, which is assumed to be strictly private in the orthodox theory, so that it includes as arguments the consumption pattern of the other individual. This modification is necessary because the type of behavior we are studying is not private but is interpersonal by definition. In many instances individuals make decisions which take other individuals, or the actions of those other individuals, into account. An individual's decision to forego his own consumption in order to make a gift to another individual is clearly one such instance. So too is the case in which the consumption bundle of one individual is influenced by the consumption bundles of other individuals. Thus, we feel that the theory of consumer choice can be generalized by the utilization of interdependent utility functions.

In the first section of this chapter we develop some standard assumptions of consumer behavior. The second section develops a two person, two good model of exchange when both individuals have strictly private utility functions. We assume there are only two goods so that we may present our analysis in graphical as well as mathematical terms. In the third section we will develop a model of exchange when our individuals have interdependent utility functions. In the fourth and last section we apply the theory of interdependent utility functions to analyze the eleemosynary behavior of one individual when the other suffers a disastrous event which reduces his holdings of consumable goods.

Assumptions Concerning the Individual Consumer[1]

In this section some assumptions will be made about the individual consumer and his behavior will be analyzed given these assumptions. It is assumed that the individual acts as a rational consumer. That is, he will act so as to maximize his utility given his income constraint and any other constraints he may face and given the alternative bundles of of goods that are available to him. We assume that his consumption bundle contains a nonnegative amount of all goods produced in his society. Thus the commodity space facing the consumer is the nonnegative orthant of Euclidean n-space where n is the number of possible goods the consumer may select for his consumption bundle.

We assume that the consumer has a cardinal utility function which can, in the two good case, be expressed as

$$U = U(x_1, x_2), \tag{2.1}$$

where x_1 and x_2 are the quantities of goods X_1 and X_2 that the individual consumes and 'U' is a function that determines the amount of utility the individual receives from the consumption of x_1 units of good X_1 and x_2 units of good X_2. It is assumed that 'U' is continuous and has continuous

[1]The analysis of the two person two good exchange model follows the general model developed in many intermediate microeconomic texts. The reader is referred to [18, Part I] or [21, Chapter 2] for a more complete development.

first-order and second-order partial derivatives. We should note that 'U' is defined over some specific period of time. While of unspecified length, this time period is assumed to be long enough that all the benefits from the consumption of a good accrue to the individual in the period in which the good was consumed. It is assumed to be short enough that the preferences of the individual may be considered constant during the period.

In general some given level of utility $U=U^*$ can be obtained from many different combinations of x_1 and x_2. The first-order partial derivatives of U with respect to X_i (i=1,2) are assumed to be greater than zero:

$$\frac{\partial U}{\partial X_i} > 0; \quad i = 1, 2. \tag{2.2}$$

This implies that the individual always receives more utility as he consumes more of X_i (i=1,2). The second-order partial derivatives are assumed to be less than zero:

$$\frac{\partial^2 U}{\partial X_i^2} < 0; \quad i = 1, 2. \tag{2.3}$$

This implies that as the individual consumes more of good X_i (i=1,2) he receives less incremental utility from each additional unit of X_i consumed.

If we take the total differential of equation 2.1 we get

$$dU = \frac{\partial U}{\partial X_1} dX_1 + \frac{\partial U}{\partial X_2} dX_2. \tag{2.4}$$

In order for the consumer to maintain a constant level of utility, dU in equation 2.4 must equal zero. This implies

$$dX_1 = - \frac{\frac{\partial U}{\partial X_2}}{\frac{\partial U}{\partial X_1}} dX_2. \tag{2.5}$$

Both the numerator and denominator of the right hand term are positive by equation 2.2. Therefore if the individual's consumption of X_1 increases by one unit he must decrease his consumption of X_2 by $\frac{\partial U}{\partial X_2} / \frac{\partial U}{\partial X_1}$ units in order to maintain the same level of utility. In order to maintain a constant level of utility, then, the consumer must compensate for his increased consumption of X_1 by decreasing his consumption of X_2.

An indifference curve is defined as the locus of all possible combinations of the goods X_1 and X_2 which give rise to some constant level of utility, U*. In Figure 2.1 we measure X_1 along the horizontal axis and X_2 along the vertical axis. The curve I* shows all possible combinations of goods X_1 and X_2 which give the consumer a utility level of U*. In other words, point (x_1^*, x_2^*) gives rise to a utility level of U*, $(U^* = U(x_1^*, x_2^*))$, as does point (x_1^o, x_2^o), $(U^* = U(x_1^o, x_2^o))$. All points or bundles of goods above and to the right of I* will give the consumer a greater level of utility than any point (bundle) on I*. All points or bundles below and to the left of I* will give the consumer a lower level of utility. We assume that every point in the commodity space is on an indifference curve with each indifference curve indicating a different level of utility. The slope of the indifference curve at any point is

$$\frac{dX_2}{dX_1} = - \frac{\frac{\partial U}{\partial X_1}}{\frac{\partial U}{\partial X_2}} \tag{2.6}$$

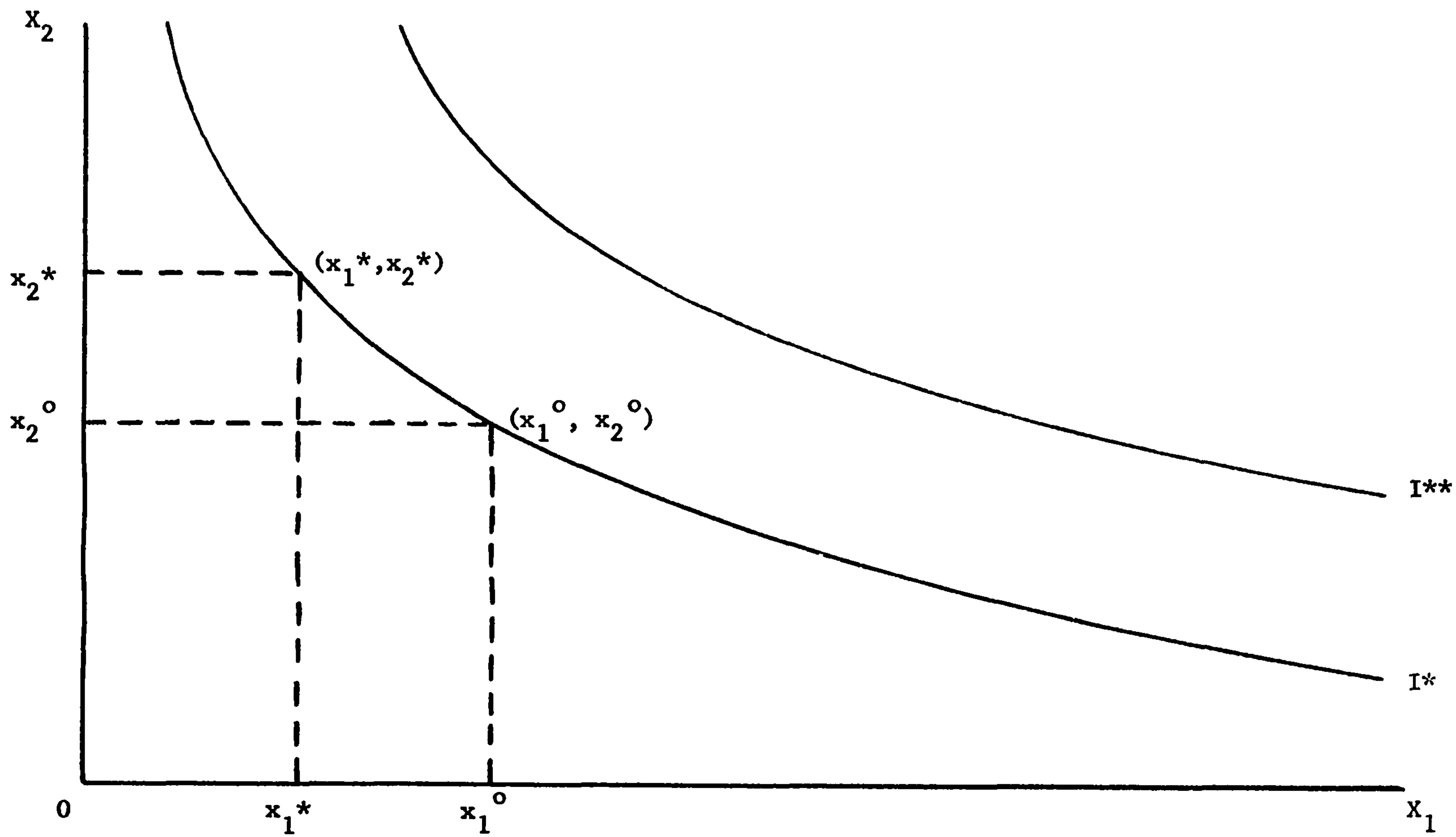

Figure 2.1. A simple indifference map

The slope of the indifference curve is called the consumer's marginal rate of substitution of X_1 for X_2 and indicates the rate at which the consumer would substitute X_1 for X_2 in order to maintain a constant level of utility.

A collection of indifference curves is called an indifference map. In Figure 2.1, I* and I** are two of the infinite number of indifference curves on the individual's indifference map. Curve I** denotes a higher level of utility than curve I*.

We will show that indifference curves cannot cross but first we need to make some assumptions concerning consumers' preferences. Let us define the commodity bundle (x_1^*, x_2^*) to be A* and the bundle (x_1^{**}, x_2^{**}) to be A**. Completeness of a consumer's preferences dictates that A* $\underline{R}$ A**, A** $\underline{R}$ A*, or both. If A* $\underline{R}$ A** and not A** $\underline{R}$ A* then A* $\underline{P}$ A**. In other words, A* $\underline{R}$ A** says that the consumer either prefers bundle A* to bundle A** or that he is indifferent between them. The same holds for A** $\underline{R}$ A* with the roles of A* and A** reversed. If both A* $\underline{R}$ A** and A** $\underline{R}$ A* hold at the same time, then the consumer is indifferent between the bundles A* and A**. In this case, both A* and A** would lie on the same indifference curve. If the consumer holds bundle A* as preferred or indifferent to bundle A** (A* $\underline{R}$ A**) but A** is not preferred or indifferent to A* (not A** $\underline{R}$ A*) then obviously the bundle A* must be strictly preferred to bundle A** (A* $\underline{P}$ A**) and A* would lie on a higher indifference curve than A**.

We assume that a consumer's preferences are transitive. Let us define the commodity bundle (x_1^o, x_2^o) to be A^o. If A* $\underline{R}$ A** and

$A^{**}\ \underline{R}\ A^o$, it necessarily follows that $A^*\ \underline{R}\ A^o$. If either $A^*\ \underline{P}\ A^{**}$ and $A^{**}\ \underline{R}\ A^o$ <u>or</u> $A^*\ \underline{R}\ A^{**}$ and $A^{**}\ \underline{P}\ A^o$, then $A^*\ \underline{P}\ A^o$.

We assume that a consumer's preferences are reflexive. That is, we assume that $A^*\ \underline{R}\ A^*$. A bundle of goods is at least as preferred as itself.

We assume that the consumption set the individual faces forms a convex set. That is, if A^* and A^{**} are both available to the consumer, then A^o is also available to the consumer where $A^o = \alpha A^* + (1-\alpha)A^{**}$ with $\alpha \in [0,1]$.

Lastly, consider any bundle A^*. We assume that the set of bundles preferred or indifferent to A^* is closed and that the set of bundles not preferred to A^* is closed. This condition ensures that the preferences of the consumer are continuous.

Given these conditions it can be shown that the consumer's indifference curves cannot cross. Consider the points A^*, A^{**}, and A^o in Figure 2.2. Each point on the indifference map corresponds to some specific level of utility. Point A^* corresponds to a utility level of U^*; point A^{**} to U^{**}; and point A^o to U^o. Since we have assumed that $\frac{\partial U}{\partial X_1} > 0$ (i=1,2) (equation 2.2) and since the point A^o has more of both X_1 and X_2 than point A^*, it must follow that $U^o > U^*$. Now since A^* and A^{**} lie on the same indifference curve, $U^* = U^{**}$ (i.e. $A^*\ \underline{R}\ A^{**}$ and $A^{**}\ \underline{R}\ A^*$). A^{**} and A^o also share a common indifference curve ($A^{**}\ \underline{R}\ A^o$ and $A^o\ \underline{R}\ A^{**}$) so U^{**} must equal U^o. By our assumption of transitivity if $A^*\ \underline{R}\ A^{**}$ and $A^{**}\ \underline{R}\ A^o$ then $A^*\ \underline{R}\ A^o$. If the consumer is indifferent between A^* and A^{**} and is also indifferent between A^{**} and A^o, then he

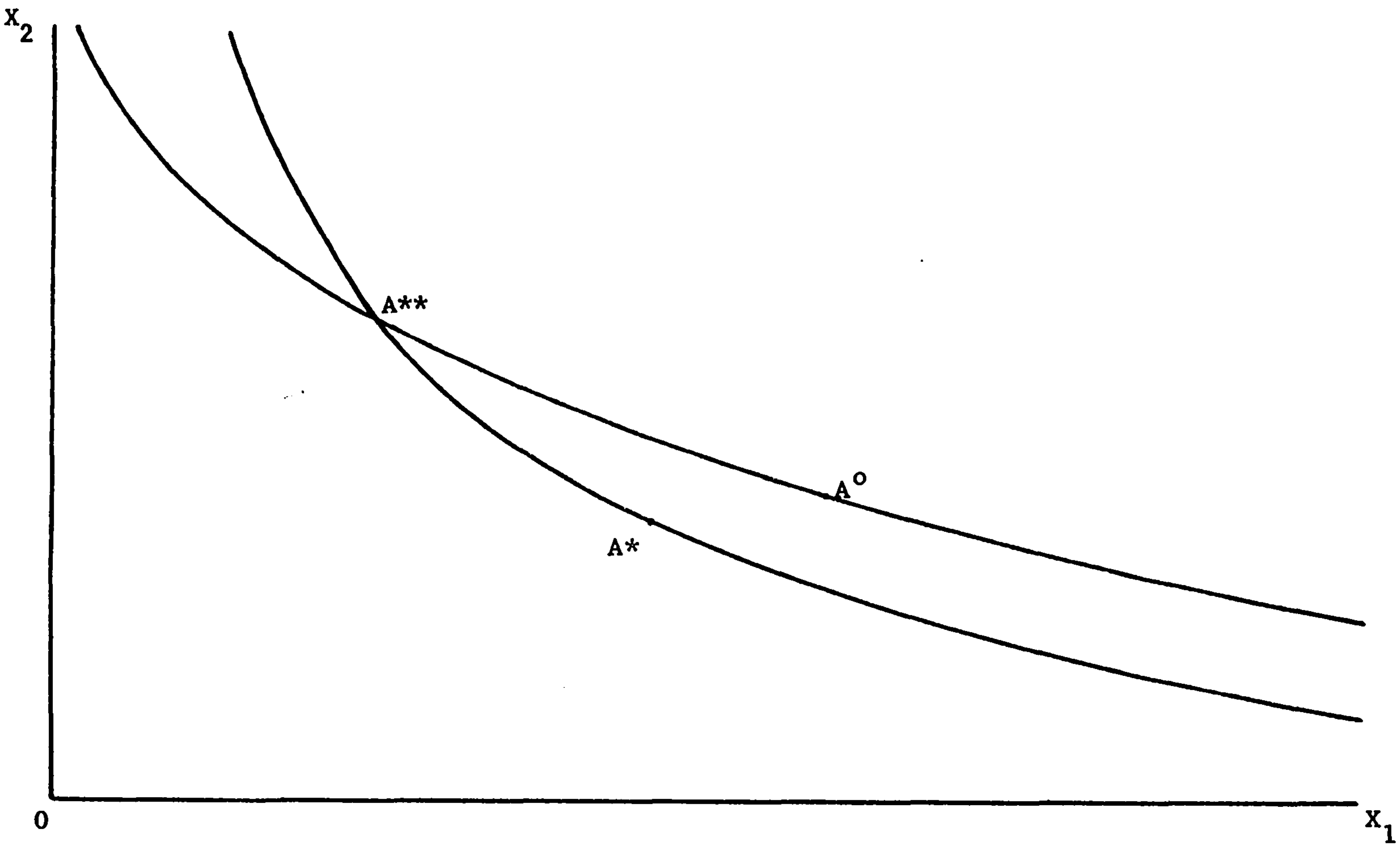

Figure 2.2. Intersecting indifference curves

must be indifferent between A* and A^o. This indicates that A* and A^o lie on the same indifference curve ($U^* = U^o$). But here we have a contradiction for $U^* < U^o$ as long as equation 2.2 holds. Since we will continue to assume that that equation holds, U* cannot equal U^o and indifference curves cannot cross.

Given that the consumer has an indifference map which indicates the level of utility he receives from various bundles of goods, it is necessary to find some method which will determine that bundle of goods which gives the consumer the greatest level of utility. If there were no constraints on the amount of goods he could consume in a given time period (all goods had a zero price) and if the consumer were never sated by any bundle of goods, then there would be no limit to the amount of utility he could receive in a single period unless there were a constraint on the supply of all goods.

Throughout the rest of this section and in the next section it will be assumed that the consumer is not satiated by any bundle of goods. The assumption will also be made that the consumer faces a budget constraint equal to his income in each period. The level of consumption the consumer can undertake is limited by this budget constraint which is written as

$$M = p_1x_1 + p_2x_2. \tag{2.7}$$

M is the individual's income and p_i is the price of good X_i (i=1,2). In general

$$M \geq \sum_{i=1}^{n} p_ix_i. \tag{2.8}$$

We will assume that the consumer spends his entire income so that

$$M \equiv \sum_{i=1}^{n} p_i x_i. \tag{2.9}$$

The utility maximizing bundle of goods must satisfy both the utility function and the budget constraint. To find this utility maximizing bundle of goods we set up the problem of Lagrange:

$$L = U(x_1,x_2) + \lambda(M - p_1x_1 - p_2x_2). \tag{2.10}$$

Differentiating L with respect to X_1, X_2, and λ and setting the resulting partial derivatives equal to zero gives

$$\frac{dL}{dX_1} = \frac{\partial U}{\partial X_1} - \lambda p_1 = 0, \tag{2.11}$$

$$\frac{dL}{dX_2} = \frac{\partial U}{\partial X_2} - \lambda p_2 = 0, \tag{2.12}$$

$$\frac{dL}{d\lambda} = M - p_1x_1 - p_2x_2 = 0. \tag{2.13}$$

Letting

$$MU_i = \frac{\partial U}{\partial X_i}, \tag{2.14}$$

it is easily observable that

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2} \quad \text{which implies} \quad \frac{MU_1}{p_1} = \frac{MU_2}{p_2}. \tag{2.15}$$

Thus the first-order conditions state that at the utility maximizing point the ratio of the consumer's marginal utilities (his marginal rate of substitution) must equal the ratio of the prices he faces. Further, utility maximization will occur at that point where the consumer spends all his income for that period. To insure that the first-order conditions lead to a maximum, it is sufficient that the principal minors of the Hessian alternate in sign with the first sign being negative (i.e., the Hessian must be negative-definite). In our two good constrained maximization problem the Hessian is

$$|H| = \begin{vmatrix} \frac{\partial^2 L}{\partial X_1^2} & \frac{\partial^2 L}{\partial X_1 \partial X_2} & \frac{\partial^2 L}{\partial X_1 \partial \lambda} \\ \frac{\partial^2 L}{\partial X_1 \partial X_2} & \frac{\partial^2 L}{\partial X_2^2} & \frac{\partial^2 L}{\partial X_2 \partial \lambda} \\ \frac{\partial^2 L}{\partial \lambda \partial X_1} & \frac{\partial^2 L}{\partial \lambda \partial X_2} & \frac{\partial^2 L}{\partial \lambda^2} \end{vmatrix} \tag{2.16}$$

The required signs for the principal minors are

$$|H_1| = \left| \frac{\partial^2 L}{\partial X_1^2} \right| < 0, \tag{2.17}$$

$$|H_2| = \begin{vmatrix} \frac{\partial^2 L}{\partial X_1^2} & \frac{\partial^2 L}{\partial X_1 \partial X_2} \\ \frac{\partial^2 L}{\partial X_1 \partial X_2} & \frac{\partial^2 L}{\partial X_2^2} \end{vmatrix} > 0; \text{ given } |H_1| < 0, \tag{2.18}$$

$$|H_3| = |H| < 0; \text{ given } |H_1| < 0 \text{ and } |H_2| > 0. \tag{2.19}$$

These second-order conditions insure that the indifference curves are convex from below. Thus the tangency between the consumer's budget constraint and an indifference curve (point A in Figure 2.3) will indicate a utility maximum and not a utility minimum. At his utility maximizing point the consumer will be consuming x_1 units of good X_1 and x_2 units of good X_2. If the consumer were at any point other than A on his budget constraint then the marginal rate of substitution of X_1 for X_2

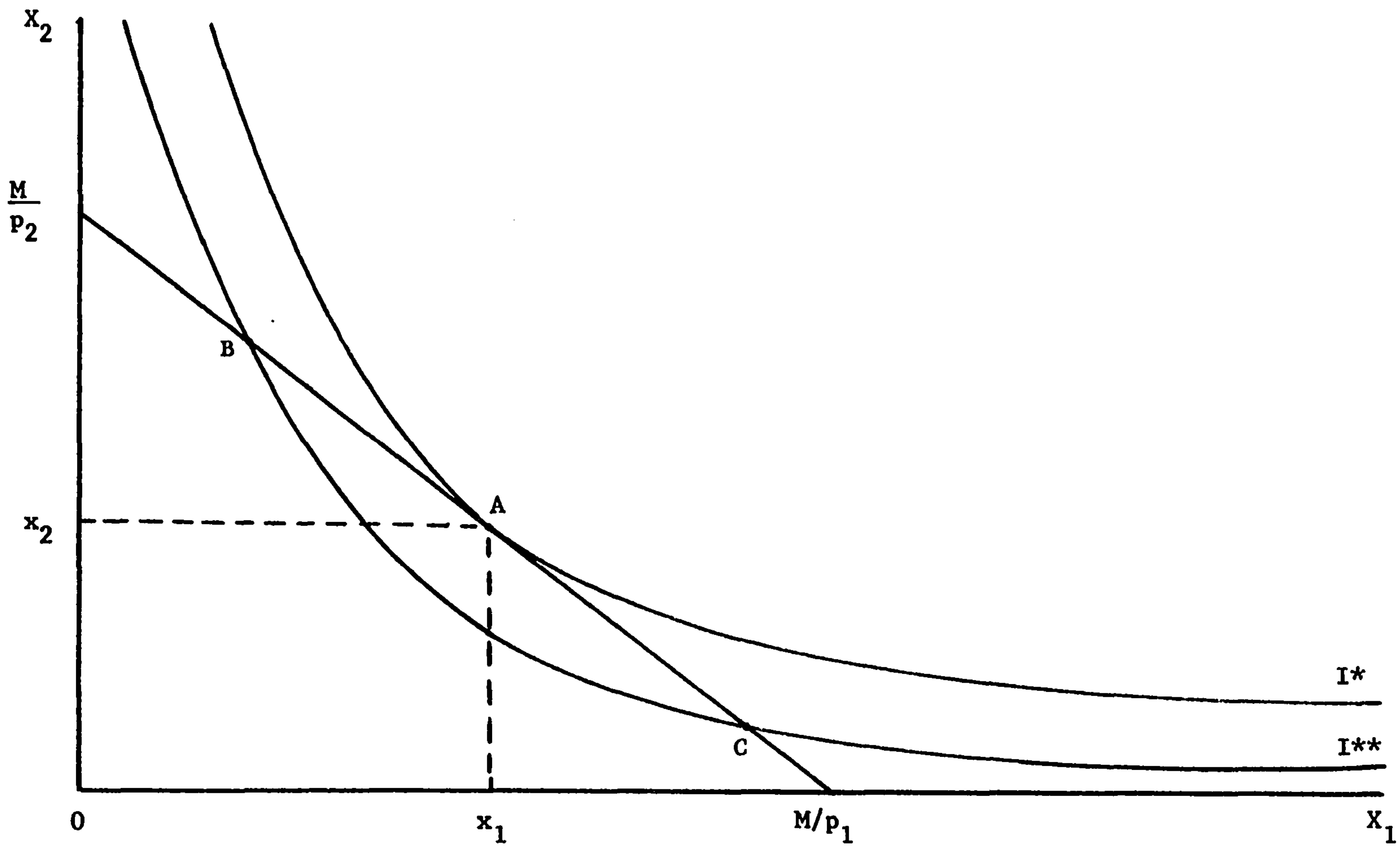

Figure 2.3. Maximizing utility given a budget constraint

would not equal the ratio of prices for the two goods. The consumer could increase his utility by increasing his consumption of the good whose marginal utility divided by its price was greatest and decreasing his consumption of the other good to the point where equation 2.15 held.

Graphically, if the consumer were purchasing bundle B or bundle C in Figure 2.3, he would not be maximizing his utility since the indifference curve I** is not tangent to the particular budget line facing the consumer. If the consumer were at point B then

$$\frac{MU_1}{P_1} > \frac{MU_2}{P_2} \quad \text{(which implies that } \frac{MU_1}{MU_2} > \frac{P_1}{P_2}\text{)} \tag{2.20}$$

and the consumer could increase his utility by consuming more X_1 and less X_2. As he did so, the marginal utility of X_1 would fall and the marginal utility of X_2 would rise. He would continue to substitute X_1 for X_2 until he reached point A where equation 2.15 holds. If the consumer were initially at point C the direction of inequality in equation 2.20 would be reversed and the consumer would substitute X_2 for X_1 until he reached point A. Point A is the utility maximizing point for the consumer given his budget constraint.

It is possible that the consumer is not able to equalize his marginal rate of substitution of X_1 for X_2 with the ratio of the prices of the two goods. If for example equation 2.20 holds then the above analysis tells us that the consumer desires to substitute X_1 for X_2 in his consumption bundle. If the individual is already consuming X_1 exclusively, then he is not able to substitute X_1 for X_2 since he has no X_2 and he is constrained to having a nonnegative amount of all goods in his consumption bundle. In this case he will spend his entire income on one good, X_1. Graphically, the consumer will be at point D in Figure 2.4.

The above analysis extends easily to the n-good case. The consumer's utility function in the n-good case is given by

$$U = U(x_1, x_2, \ldots, x_n) \tag{2.21}$$

and his budget constraint by

$$M = p_1x_1 + \ldots + p_nx_n = \sum_{i=1}^{n} p_ix_i. \tag{2.22}$$

The Lagrangian function to be maximized is:

$$L = U(x_1, x_2, \ldots, x_n) + \lambda(M - \sum_{i=1}^{n} p_ix_i). \tag{2.23}$$

For a utility maximum at an interior point the first-order conditions are:

$$\frac{dL}{dX_i} = \frac{\partial U}{\partial X_i} - \lambda p_i = 0; \qquad i = 1, 2, \ldots, n \tag{2.24}$$

and

$$\frac{dL}{d\lambda} = M - \sum_{i=1}^{n} p_ix_i = 0. \tag{2.25}$$

By taking ratios of the equations in equation 2.24 we get a generalization of equation 2.15.

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} = \ldots = \frac{MU_n}{p_n}\,. \tag{2.26}$$

It is assumed, of course, that the Hessian is negative-definite for a utility maximum.

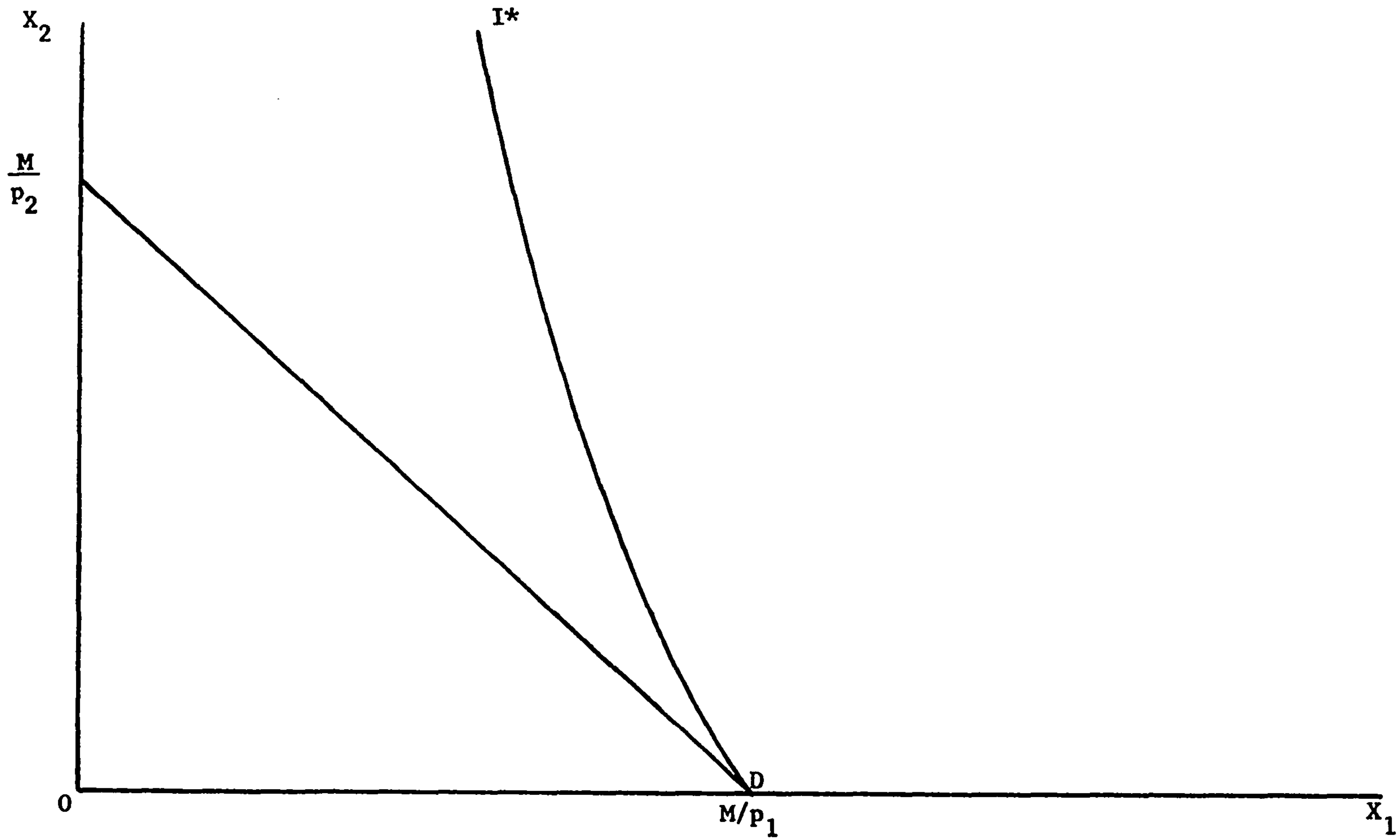

Figure 2.4. A utility maximum when only one good is consumed

For a utility maximum at a boundary the first-order conditions are:

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} = \ldots = \frac{MU_i}{p_i} \geq \frac{MU_{i+1}}{p_{i+1}} \geq \ldots \geq \frac{MU_n}{p_n} \tag{2.27}$$

and

$$M = \sum_{i=1}^{n} p_i x_i . \tag{2.28}$$

Again the Hessian is assumed to be negative-definite. In this case the individual will consume positive amounts of the first i goods and zero amounts of the remaining n-i goods.

A Two Person Model of Exchange

This section outlines a simple two person model of exchange. The individuals are called individual A and individual B. Each of these individuals is assumed to have a strictly private utility function:

$$U_A = U_A(x_1^A, x_2^A) \text{ and } U_B = u_B(x_1^B, x_2^B). \tag{2.29}$$

It is initially assumed that there are two goods, X_1 and X_2. These two goods are assumed to be in fixed supply, $X_1 = \bar{X}_1$ and $X_2 = \bar{X}_2$. The supply of the two goods is distributed between individual A and individual B so that $x_1^A + x_1^B = \bar{X}_1$ and $x_2^A + x_2^B = \bar{X}_2$.

Since the amounts of X_1 and X_2 are fixed for the time period under consideration, the maximum amount that either individual can consume is $\bar{X}_1$ units of X_1 and $\bar{X}_2$ units of X_2. In Figure 2.5a we have drawn the relevant area of consumer A's commodity space. It is a rectangle of dimensions $0_A\bar{X}_1$ by $0_A\bar{X}_2$. Individual B's relevant commodity space has the same dimensions (Figure 2.5b).

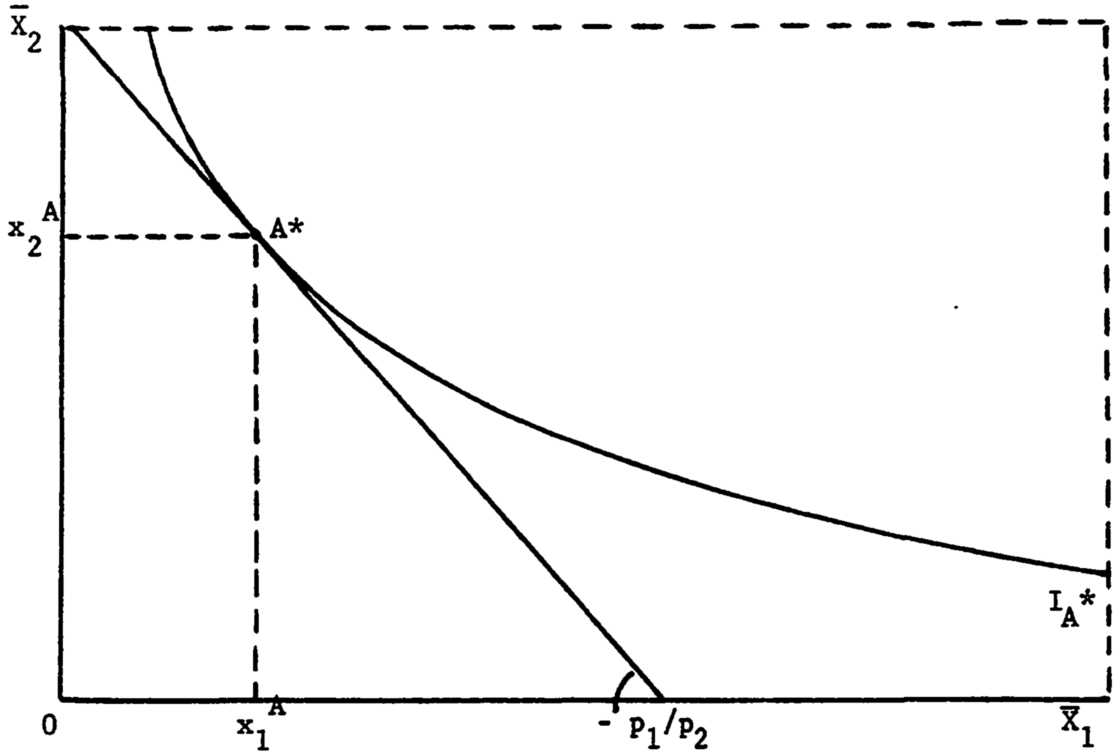

Figure 2.5a. Individual A's commodity space

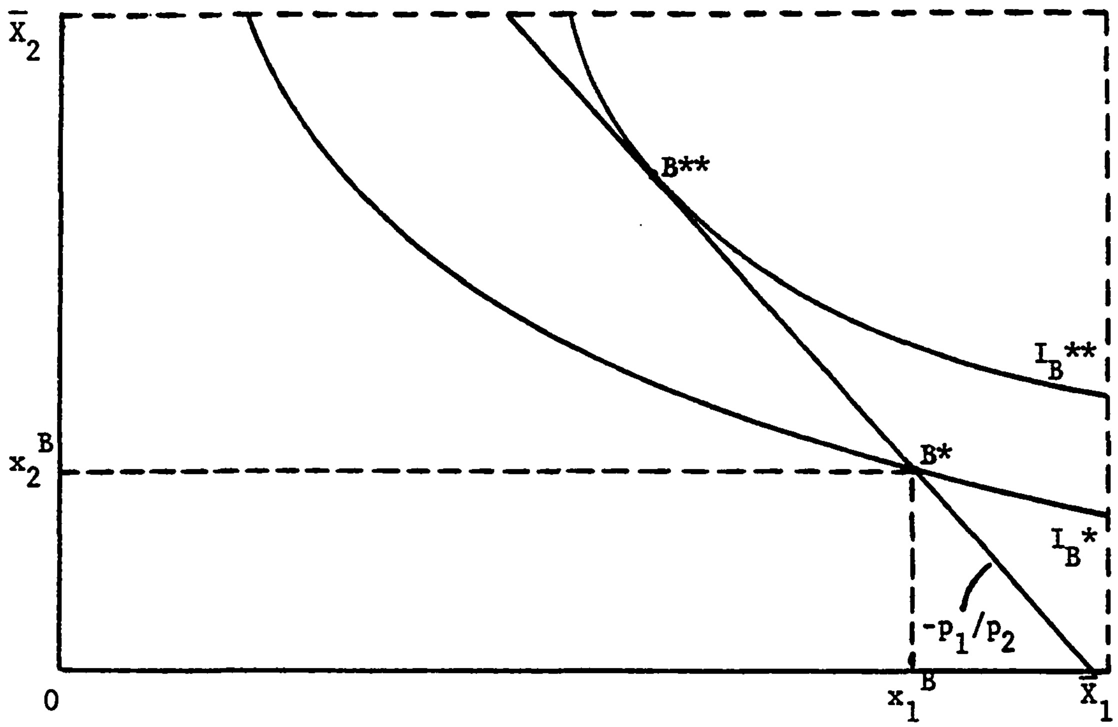

Figure 2.5b. Individual B's commodity space

The manner by which goods are produced, how much of each is produced, and how the supply of goods is initially allocated is not of interest here. Our concern is only with the movement from the initial distribution to the final distribution.

The initial distribution of the two goods is assumed to give A x_1^A units of X_1 and x_2^A units of X_2 (point A* in Figure 2.5a). B will initially have $x_1^B = \bar{X}_1 - x_1^A$ units of X_1 and $x_2^B = \bar{X}_2 - x_2^A$ units of X_2 (point B* in Figure 2.5b). Point A* lies on A's indifference curve labeled I_A* and point B* lies on B's indifference curve I_B*. If there is some "central authority" that has the power to set relative prices for X_1 and X_2 (p_1/p_2) then there will be a line through both points A* and B* with the slope ($-p_1/p_2$). If these price lines are tangent to both I_A* at point A* and I_B* at point B* then both A and B will be maximizing their utility and there will be no incentive for them to trade with each other. If, however, the price line is not tangent to I_A* or/and I_B* then there will be incentive for one or both individuals to initiate trade. Each will act out of the purely selfish motive of maximizing his own utility.

In Figures 2.5a and 2.5b it is seen that at the price ratio p_1/p_2 individual A is maximizing his utility while individual B is not. At the price ratio p_1/p_2 individual B could increase his utility by consuming more X_2 and less X_1. At the given price ratio and his initial bundle of goods (which together define his income constraint), B's utility maximizing point is given by point B**. Point B** is the tangency point between the price line that runs through point B* and B's indifference curve

I_B**. B desires to move from point B* to point B**. In the two person case, the only way he can achieve this movement is through trade with individual A. Since A is already maximizing his utility given the price ratio and his initial allocation he will refuse to trade at the given price ratio. If neither A nor B is able to adjust (or influence the adjustment of) the relative prices then A will keep his initial allocation A* and B will keep his initial allocation B* even though B would prefer to be at point B**. In this case there will be an excess supply of good X_1 equal to the difference between B's holdings of X_1 at B* and B**. Likewise there will be an excess demand for good X_2 equal to the difference in B's actual holdings of X_2 at B* and his desired holdings at B**.

This can be seen more clearly if the two graphs (Figures 2.5a and 2.5b) are combined into one. If Figure 2.5b is rotated 180 degrees and superimposed upon Figure 2.5a we get Figure 2.6. Figure 2.6 is commonly called the Edgeworth box in economic literature. The dimensions of the box are the same as those in Figures 2.5a and 2.5b, $\bar{X}_1$ by $\bar{X}_2$. The origin for individual A is 0_A in the southwest corner of the box and that for individual B is 0_B in the northeast corner. A's level of utility increases as he moves in a northeasterly direction and B's increases as he moves in a southwesterly direction. A's initial allocation is as before (x_1^A and x_2^A) as is B's ($x_1^B = \bar{X}_1 - x_1^A$ and $x_2^B = \bar{X}_2 - x_2^A$). Thus points A* and B* are the single point - point C. As before I_A* and I_B* are the indifference curves that contain point C.

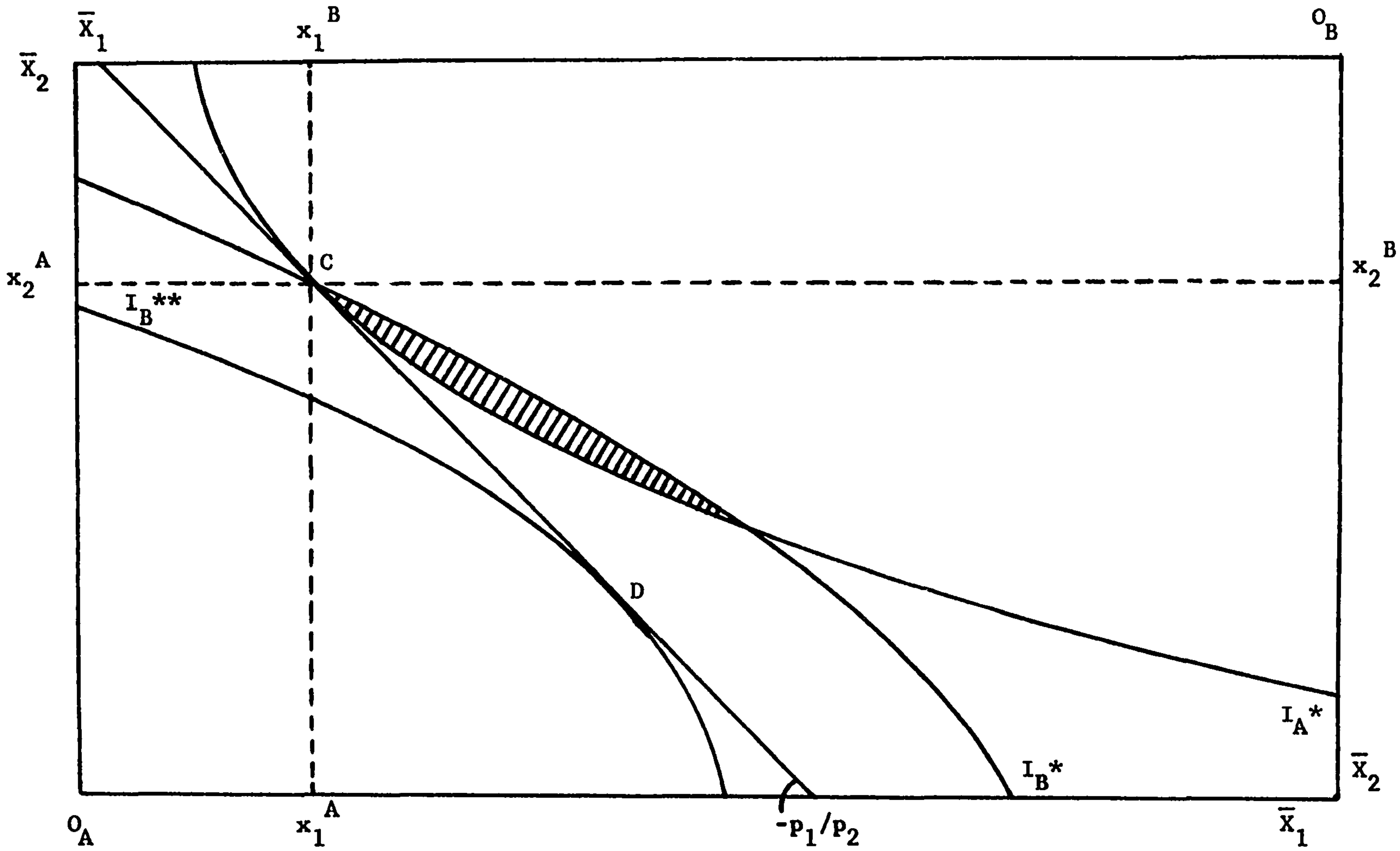

Figure 2.6. The Edgeworth box

The original price line $(-p_1/p_2)$ is drawn so as to be tangent to I_A* at C as before. At this price ratio A is maximizing his utility while B would maximize his at point D (equal to point B** in Figure 2.5b). Clearly a move from C to D is in B's best interest. It is just as clear, however, that A will not be willing to make the move since the movement from C to D will leave him on an indifference curve lower than the one he has already attained. In order for A to be willing to trade with B, the final distribution must be on or to the northeast of I_A*. Likewise B will not trade unless he ends up on or to the southwest of I_B*. Therefore, trade will only take place in the shaded area of Figure 2.6 bounded by I_A* and I_B*.

In order that the final distribution be a Pareto efficient distribution it is necessary that both consumers (traders) equate their marginal rates of substitution of X_1 for X_2 with the ratio of prices for the two goods. Both consumers must face the same set of relative prices and the final distribution must be such that $x_1^A + x_1^B = \bar{X}_1$ and $x_2^A + x_2^B = \bar{X}_2$. This tells us that at the point of a Pareto efficient distribution the marginal rates of substitution of X_1 for X_2 for the two consumers are the same. Their indifference curves are tangent at points of Pareto efficient distribution. The locus of these tangency points is called the contract curve. The final distribution, if it is to be efficient, must lie on the contract curve. Further, the final distribution is restricted to lie on that portion of the contract curve which is bounded by the two indifference curves that pass through the initial distribution.

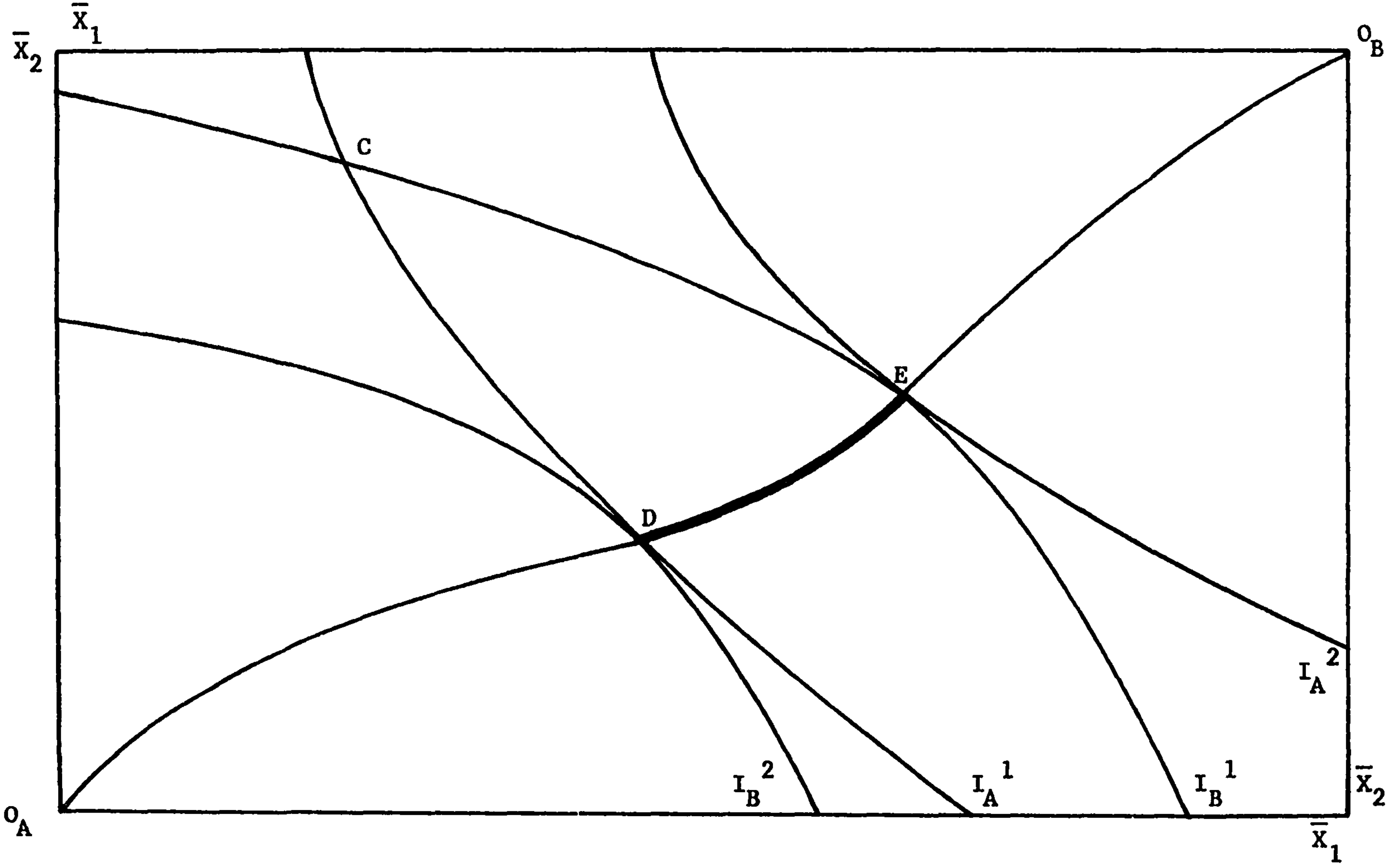

Figure 2.7. The contract curve

In Figure 2.7 the initial distribution of X_1 and X_2 is at point C. The contract curve is $0_A DE0_B$. The Pareto relevant portion of the contract curve is DE since along this portion neither trader is made worse off by a trade from point C. Individual A would like to be as close to point E as possible since point E maximizes his utility given that B's utility must not fall below its initial level represented by indifference curve $I_B^{\ 1}$. Individual B would naturally like to be as close to point D as possible for a similar reason. The final, post-trade distribution could be anywhere along the DE segment of the contract curve. Its exact location will depend upon several factors. One is the relative bargaining strengths of the two individuals. The individual with the greatest bargaining power will reap most of the gains from trade.

Another factor is the number of separate trades that take place before the contract curve is reached. If the move from point C to the contract curve is made in one move, it will be necessary for the relative price line, which defines the terms of trade, to be tangent to each of the relevant indifference curves and to pass through the point C. This condition is shown in Figure 2.8 where point C is the initial distribution, point F is the final distribution (on the contract curve) and $-p_1/p_2$ is the ratio of prices or terms of trade. The price line is tangent to both $I_A^{\ 2}$ and $I_B^{\ 2}$ at point F and passes through point C. If there is only one set of relative prices that gives rise to a one-move final distribution on the contract curve (there may be several or there may be none), then, given that it takes only one trade to reach the final

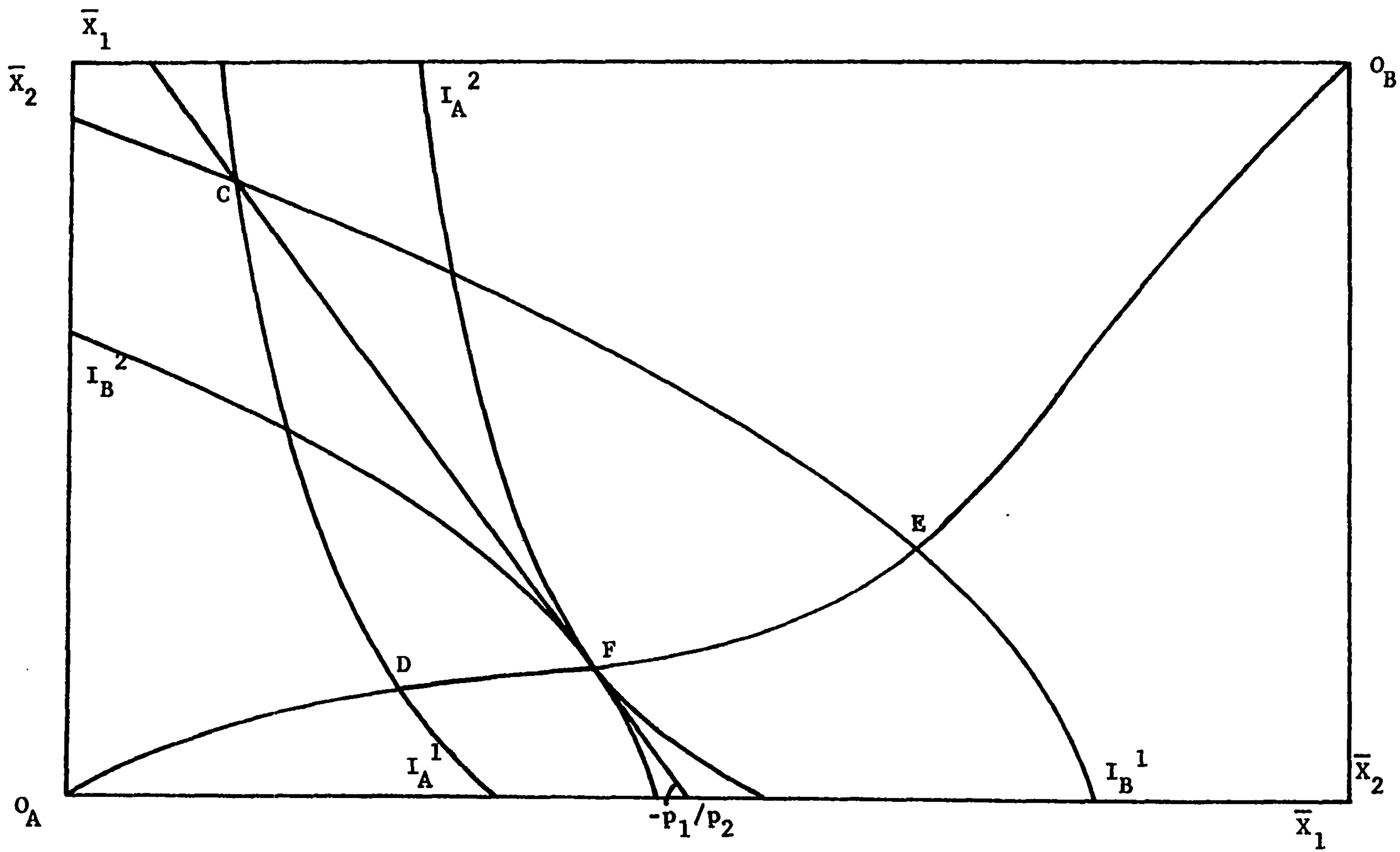

Figure 2.8. A one trade move to the contract curve

distribution from the initial distribution, the final distribution will be uniquely determined. In one-trade situations, the bargaining strengths of the individuals will not enter unless there is more than one set of relative prices which will give rise to an efficient final distribution.

If it takes more than one trade to reach the contract curve from the initial distribution, then the final distribution will depend, in part, upon the sequence of trades. Each trade that does not lead to a Pareto efficient distribution will reduce the portion of the contract curve that is relevant for future trades. In Figure 2.9 the initial distribution is at point C and the portion of the contract curve that is of relevance before any trade takes place is DE. Let the first trade between A and B be inefficient in the sense that the distribution obtained is not a point on the (DE segment of the) contract curve. The terms of trade $(-p_1/p_2)$ lie within the trading area so that there is an incentive for the two traders to make a trade. In general, given the terms of trade, one trader will reach a utility maximum before the other. In Figure 2.9, given the initial distribution and the terms of trade, trader B will reach a utility maximum at point G. Once he has reached point G, individual B will refuse to make any further exchanges at the original terms of trade even though individual A desires to continue exchanging X_2 for X_1. Point G becomes the new starting point and the new trade area is the intersection of each individual's "no worse than G set" (area KGL). The relevant portion of the contract curve shrinks from DE to HJ. A new set of relative prices (terms of trade) is agreed upon and trading continues until a distribution on the contract curve is reached.

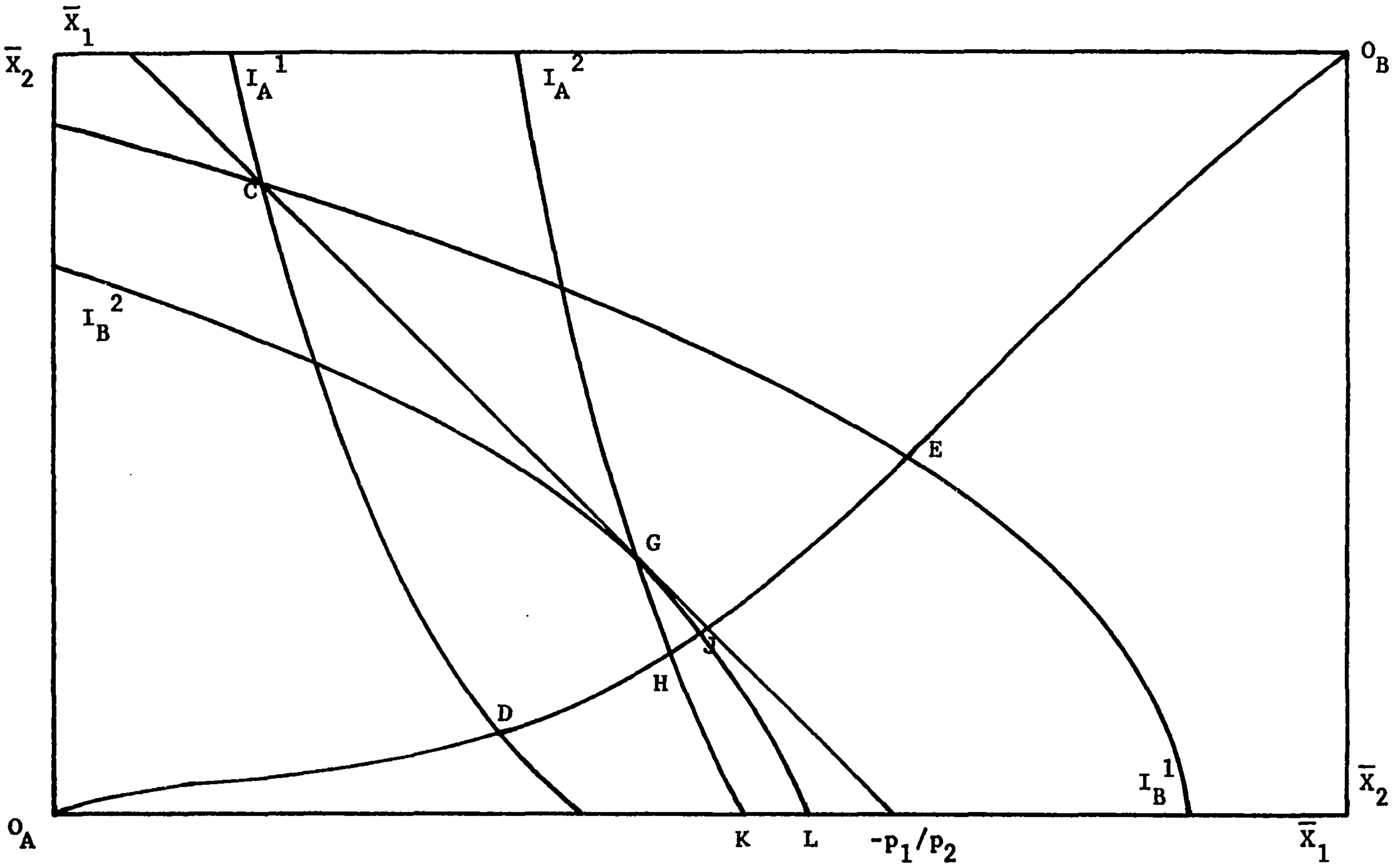

Figure 2.9. Movement to the contract curve when more than one trade is made

If part of the contract curve coincides with one of the boundaries (see Figure 2.10) then, in general, the marginal rates of substitution will not be the same for the two individuals along this portion of the contract curve. In Figure 2.10 the contract curve is $0_A GFED0_B$ and the relevant portion, given an initial distribution at point C is DEF. Between D and E the contract curve coincides with the boundary indicating a zero consumption of good X_2 by trader B. Along this segment (DE) the individuals' marginal rates of substitution of X_1 for X_2 should not be equal except at point E. At all points on DE other than E, the marginal rate of substitution of X_1 for X_2 will be less for individual A than for individual B. If the contract curve coincides with another boundary, then the marginal rate of substitution of X_1 for X_2 might be greater for individual A than for individual B.

If we extend the analysis presented in this section to the n-good case (with $n > 2$) the results will remain basically the same. If an interior solution exists, then there will be at least one set of exchange ratios (set of prices) which will lead to this interior solution. For this interior solution the two traders will still equate their marginal rates of substitution for any two commodities with the exchange ratios for these commodities. If a solution lies on one of the boundaries of the restricted n-space consumption set, then one of the traders is consuming all of the relevant good and the other trader is not consuming any of that good. Let us define a "bounded" good as a good which is not consumed by one of the consumers. When bounded goods are present the two traders will continue to equate their marginal rates of substitution for

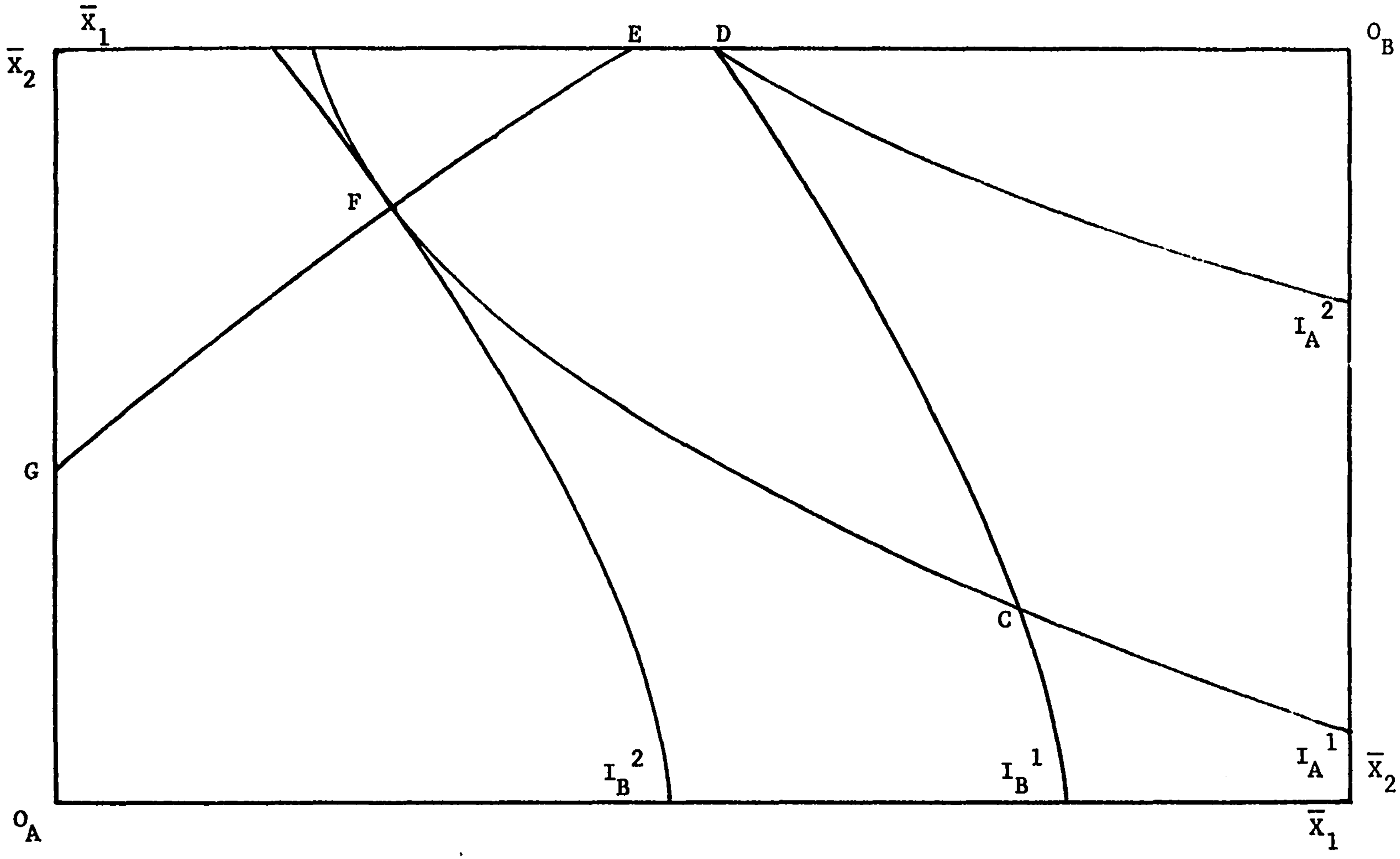

Figure 2.10. The contract curve coincides with a boundary of the consumption set

each pair of the "nonbounded" goods with the relative prices (terms of trade) for those goods. The relative prices of the "bounded" goods and any other good must fall between the marginal rates of substitution of the two traders for the two goods if a Pareto efficient point on the boundary of the consumption set is to be reached. The only exception would be when the indifference surfaces of the two traders are just tangent at the boundary. This would occur at a point such as point E in Figure 2.10. When this occurs the ratio of prices for the "bounded" and "nonbounded" goods must equal both traders' marginal rates of substitution for those two goods.

This section has been concerned with exchange in a two person world. In this section the two individuals were assumed to have strictly private utility functions, a restriction which will be relaxed in the next section.

A Two Person Model of Exchange When Utility Functions are Interdependent[1]

In this section we will present a two person, two good exchange model when the utility function of at least one individual exhibits interdependencies related to the other individual. We will begin by analyzing the case in which only one of the individuals has an interdependent utility function and then move to the case where both individuals have interdependent utility functions.

As in the previous section we are concerned only with how individuals will redistribute their initial allocation of goods. We are not looking

[1]This section has been influenced by [4; 5; 12; 13; 41; 45].

at the production side of the market and are not here interested in how the initial distribution came about. In order to simplify the mathematics we will assume that there are only two goods in our hypothetical world - good X_1 and good X_2. Each of these two goods is assumed to be a homogeneous product that is in fixed supply. The total amount of each good $\bar{X}_1$ and $\bar{X}_2$ is divided between our two individuals - individual A and individual B. Individual A has x_1^A units of X_1 and x_2^A units of X_2; B has x_1^B units of X_1 and x_2^B units of X_2. We have already stated the constraints that $x_1^A + x_1^B = \bar{X}_1$ and $x_2^A + x_2^B = \bar{X}_2$. We assume that the tastes of the two individuals are given for the period under consideration. We assume that exchanges and transfers can be made costlessly and we assume nonappropriation - an individual cannot be forced to take actions not in his own best interest.

Given the above assumptions and the fact that only one of the two individuals (say individual A) has an interdependent utility function, we may write the utility function of the two individuals as:

$$U_A = U_A(X_1^A, X_2^A; X_1^B, X_2^B; U_B); \tag{2.30}$$

and

$$U_B = U_B(X_1^B, X_2^B). \tag{2.31}$$

The fact that B's consumption of goods X_1 and X_2 and his utility function U_B enter A's utility function indicates that B's consumption behavior may influence A in two distinct ways. First, B's consumption of X_1 and X_2 may directly influence A's utility. If such an influence exists we say a "goods" externality is present. $\partial U_A/\partial X_i^B \neq 0$; $i = 1$ and/or 2 indicates that a "goods" externality is present. Second, B's consumption of X_1 and X_2 may indirectly influence A's utility. In this instance A will be influenced by B's general level of utility without regard to how B attains that level. We call this type of influence a "utility" externality and denote it by $\partial U_A/\partial U_B \neq 0$. "Goods" and "utility" externalities may occur together ($\partial U_A/\partial X_i^B \neq 0$, $i = 1$ and/or 2, and $\partial U_A/\partial U_B \neq 0$) or separately ($\partial U_A/\partial X_i^B = 0$, $i = 1, 2$ and $\partial U_A/\partial U_B \neq 0$ <u>or</u> $\partial U_A/\partial X_i^B \neq 0$, $i = 1$ and/or 2 and $\partial U_A/\partial U_B = 0$). If both externalities are nonexistent ($\partial U_A/\partial X_i^B = 0$, $i = 1, 2$ and $\partial U_A/\partial U_B = 0$) then the situation is one in which both individuals have strictly private utility functions. This situation was discussed in the preceding section. The distinction between a "goods" externality and a "utility" externality is not as important as has been assumed by others [12, pp. 3, 14]. At least it is not important in the two person case when goods are in fixed supply. We will discuss this issue later in this section.

In order to find the slopes of the indifference curves for A and B we follow the same procedure used in the first section of this chapter. For B we get the same result:

$$\frac{dX_2^B}{dX_1^B} = -\frac{\partial U_B/\partial X_1^B}{\partial U_B/\partial X_2^B}\ . \tag{2.32}$$

B's indifference curves are always negatively sloped since $\partial U_B/\partial X_i^B > 0$ ($i = 1, 2$). We assume the second order conditions hold which will ensure B's indifference curves are convex to his origin.

A's indifference curves are not so nicely behaved. The total differential of U_A is:[1]

$$dU_A = \frac{\partial U_A}{\partial X_1{}^A} dX_1{}^A + [\frac{\partial U_A}{\partial X_1{}^B} + \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_1{}^B}] dX_1{}^B + \frac{\partial U_A}{\partial X_2{}^A} dX_2{}^A$$

$$+ [\frac{\partial U_A}{\partial X_2{}^B} + \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_2{}^B}] dX_2{}^B. \quad (2.33)$$

If we assume perfect knowledge then $\partial U_B/\partial X_i{}^B$ (i=1, 2) in equation 2.33 will be the same as in equation 2.32. If we do not make this assumption, then these terms in equation 2.33 will be individual A's best estimates of B's marginal utilities. In this two person world the assumptions that all goods are in fixed supply and that all exchanges and transfers can be made costlessly indicate that $dx_1{}^A = -dx_1{}^B$ and $dx_2{}^A = -dx_2{}^B$. Thus at A's utility maximizing point

$$dU_A = [\frac{\partial U_A}{\partial X_1{}^A} - \frac{\partial U_A}{\partial X_1{}^B} - \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_1{}^B}] dX_1{}^A$$

$$+ [\frac{\partial U_A}{\partial X_2{}^A} - \frac{\partial U_A}{\partial X_2{}^B} - \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_2{}^B}] dX_2{}^A = 0. \quad (2.34)$$

The slope of A's indifference curves is given by

---

[1] The total differential of U_A is actually $dU_A = \partial U_A/\partial X_1{}^A\, dX_1{}^A + \partial U_A/\partial X_2{}^A\, dX_2{}^A + \partial U_A/\partial X_1{}^B\, dX_1{}^B + \partial U_A/\partial X_2{}^B\, dX_2{}^B + \partial U_A/\partial U_B\, dU_B$. We have substituted the value of dU_B from equation 2.4 to get our equation 2.33.

$$\frac{dX_2^{\,A}}{dX_1^{\,A}} = - \frac{\dfrac{\partial U_A}{\partial X_1^{\,A}} - \dfrac{\partial U_A}{\partial X_1^{\,B}} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_1^{\,B}}}{\dfrac{\partial U_A}{\partial X_2^{\,A}} - \dfrac{\partial U_A}{\partial X_2^{\,B}} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_2^{\,B}}} \quad . \tag{2.35}$$

The slope of A's indifference curve will be dependent upon the consumption of both goods by both individuals. Obviously if neither a "goods" externality nor a "utility" externality is present, the slope is dependent only on A's consumption of the two goods.

We will continue to assume that $\partial U_j/\partial x_i^{\,j} > 0$ where $i = 1, 2$ and $j = A, B$. The signs of $\partial U_A/\partial x_i^{\,B}$ $i = 1, 2$ and $\partial U_A/\partial U_B$ can be positive or negative. If $\partial U_A/\partial x_i^{\,B}$ $i = 1, 2$ is positive then individual A will receive an increase in his utility level when B consumes good X_i. When $\partial U_A/\partial x_i^{\,B} < 0$ increases in B's consumption of good X_i will lower A's utility. Similarly if $\partial U_A/\partial U_B > 0$ then A's knowledge that B's utility has increased will increase his own utility level. When $\partial U_A/\partial U_B < 0$ increases in B's utility will cause decreases in A's. If $\partial U_A/\partial U_B > 0$ we say that A is benevolent toward B and if $\partial U_A/\partial U_B < 0$ we say he is malevolent toward B. We will use the same terminology with regards to "goods" externalities noting that if $\partial U_A/\partial x_i^{\,B} > 0$ and $\partial U_A/\partial X_j^{\,B} < 0$ $i \neq j$ A will be both benevolent and malevolent toward B at the same time. This can cause some confusion so the terms benevolence and malevolence will be used when their meanings are unambiguous. If a situation should arise where A is both benevolent and malevolent toward B we will say A is benevolent toward B with respect to good X_i and malevolent with respect to X_j.

Since the slope of A's indifference curves are dependent upon the consumption patterns of individual B, it is no longer necessary that these curves always be negatively sloped. If all externalities facing individual A are negative (A harbors malevolent feelings toward B) then his indifference curves will always have a negative slope.

Recall that the equation for A's indifference curves is

$$\frac{dx_2^A}{dx_1^A} = - \frac{\dfrac{\partial U_A}{\partial X_1^A} - \dfrac{\partial U_A}{\partial X_1^B} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_1^B}}{\dfrac{\partial U_A}{\partial X_2^A} - \dfrac{\partial U_A}{\partial X_2^B} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_2^B}} \quad . \tag{2.35}$$

If A holds only malevolent feelings toward B both $\partial U_A/\partial x_i^B$ (i = 1, 2) and $\partial U_A/\partial U_B$ are negative. $\partial U_B/\partial x_i^B$ (i = 1, 2) is always positive, of course. This indicates that both the numerator and denominator of the term in brackets are positive which means that A's indifference curves are negatively sloped (and convex from below given that the Hessian is negative-definite as assumed).

We are concerned with A's benevolent feelings, not his malevolent feelings so we will deal solely with his benevolent feelings from this point on. If A is benevolent toward B then both $\partial U_A/\partial X_i^B$ (i=1,2) and $\partial U_A/\partial U_B$ are positive. We assume that the second-order derivatives $\partial^2 U_A/\partial X_i^{B^2}$ (i=1,2) and $\partial^2 U_A/\partial U_B^{\,2}$ are negative. If A's benevolence is strong enough then it is possible for the numerator of equation 2.35 to be negative, for the denominator to be negative, or for both to be negative. If only one of the denominator or numerator is negative then A's indifference curve will have a positively sloped section. If both are negative, A's indifference curve will be negatively sloped but will have the "wrong" curvature (see Figure 2.11).

Since B's utility function is strictly private, his indifference curves (I_B^1 and I_B^2) are negatively sloped and convex to his origin in the northwest corner of the box. We have drawn A's indifference curves (I_A^1, I_A^2, etc.) as a series of concentric circles since the presence of "goods" and "utility" externalities in his utility function may lead portions of his indifference curves to exhibit positive slopes or negative slopes of the wrong curvature. These circular indifference curves imply that A is indifferent among all distributions of X_1 and X_2 between B and himself which lie on a particular curve. The point E is A's bliss point, the point where he gains his highest level of utility. Note that point E is A's bliss point _vis-a-vis_ the goods X_1 and X_2 _and_ the other individual. It is not his bliss point _vis-a-vis_ the goods X_1 and X_2 alone. If we exchanged individual B with an individual C and left the amounts of the two goods constant at $\bar{X}_1$ and $\bar{X}_2$, then A's bliss point would move unless A felt exactly the same about C as he felt about B. The donor individual (the individual experiencing the externalities) will not be indifferent to the object individual or to the characteristics of the object individual. The identity of the other fellow, his personality, behavior patterns and other personal characteristics will influence the degree of benevolence shown by the donor individual. In general, the more benevolent A feels toward the other individual the closer to A's origin will be A's bliss point. It is not, however, necessary for A's bliss point to fall within the boundaries of the Edgeworth box.

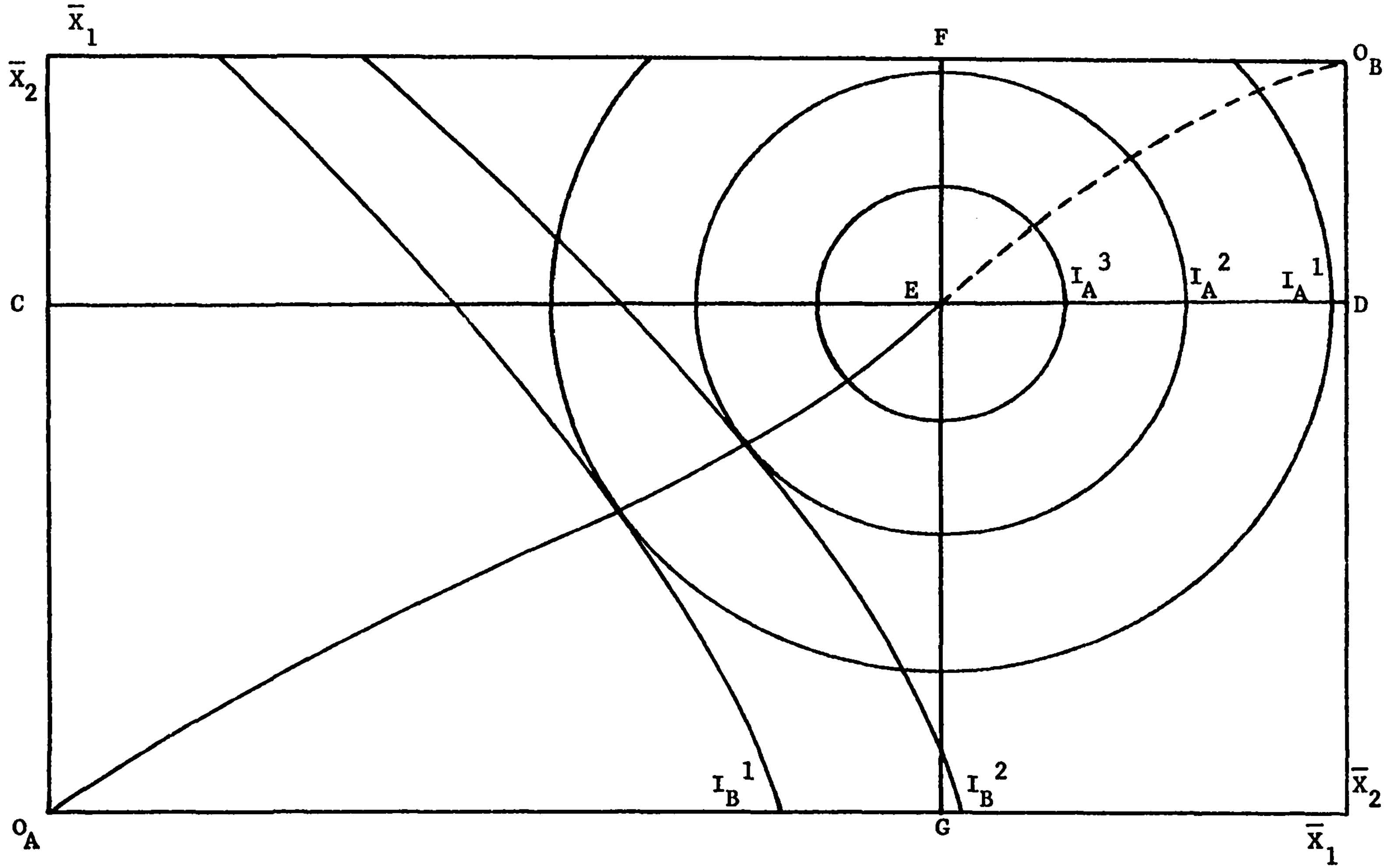

Figure 2.11. An indifference map when preferences are interdependent

The ridge lines CED and FEG cross A's indifference curves at all points where the slope is either infinite or zero respectively. These ridge lines divide the Edgeworth box into four conceptually distinct regions. The region bounded by O_ACEG will be called the region of pure exchange. Any initial distribution of goods within this region will lead to a point on the contract curve (O_AE) by a process described in the previous section of this chapter.

The regions bounded by CEF and DEG are regions of possible charity. Within these regions A may increase his utility by making a unilateral transfer to B of one of the goods (X_2 in region CEF and X_1 in region DEG). The region bounded by FED is the region of charity. If the initial distribution is within this region, A can always reach his bliss point by transferring some of each good to B.

Let us consider a region of possible charity. Given that the initial distribution is at point L in Figure 2.12, A would prefer to move from point L to point E since his utility is maximized at point E. He cannot reach E, however, since any movement he makes from point L is constrained to lie to the left of B's indifference curve through L (I_B^1). At best he can reach the point P on the contract curve and B's indifference curve I_B^1.

It might seem that the relevant portion of the contract curve is KP, the region between the two indifference curves through L. However, if A is a utility maximizer as we suppose, he will realize that he need be at no point to the left of M and that the relevant portion of the contract curve is MP. Given the initial distribution at L, A can

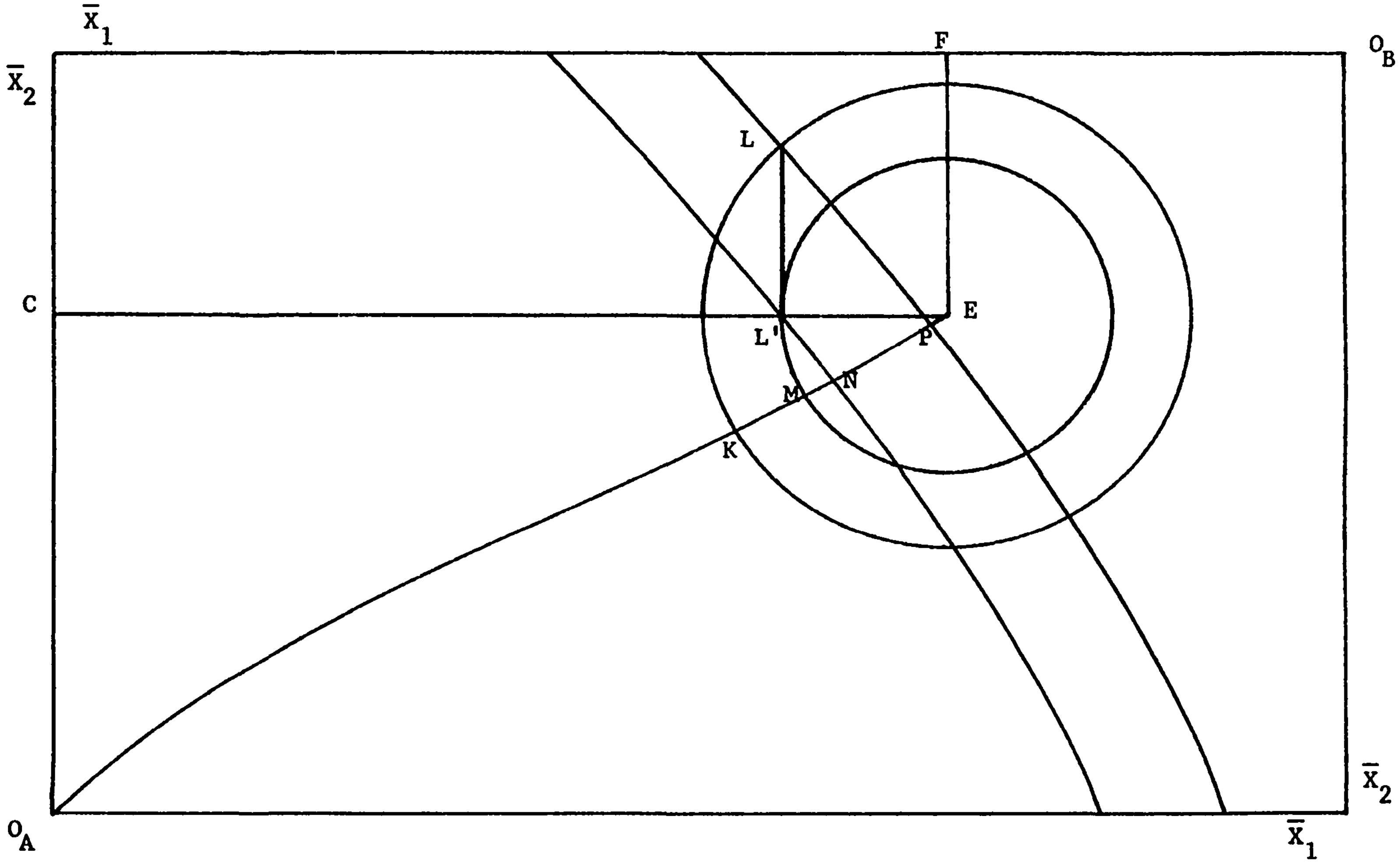

Figure 2.12. Movement in a region of possible charity

increase his utility by means other than exchange. He can give away some of the good X_2 and be better off than he was before.

Recall that the first order condition for A to maximize his utility is given by equation 2.34 when $dx_1^A = - dx_1^B$ and $dx_2^A = - dx_2^B$. Now if we hold the distribution of X_1 constant, A will maximize his utility when

$$\frac{dU_A}{dX_2^A} = \frac{\partial U_A}{\partial X_2^A} - \frac{\partial U_A}{\partial X_2^B} - \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_2^B} = 0. \qquad (2.36)$$

Since L is on that particular positively sloped portion of A's indifference curve shown in Figure 2.12, we know that

$$\frac{dU_A}{dX_2^A} = \frac{\partial U_A}{\partial X_2^A} - \frac{\partial U_A}{\partial X_2^B} - \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_2^B} < 0 \qquad (2.37)$$

and

$$\frac{dU_A}{dX_1^A} = \frac{\partial U_A}{\partial X_1^A} - \frac{\partial U_A}{\partial X_1^B} - \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_1^B} > 0. \qquad (2.38)$$

A can increase his utility by giving X_2 to individual B. As he gives X_2 to B $\partial U_A/\partial X_2^A$ will rise since A has less of good X_2, $\partial U_B/\partial X_2^B$ will fall since B has more of it, and $\partial U_A/\partial X_2^B$ and $\partial U_A/\partial U_B$ will fall given our assumption that the second derivatives of both "goods" and "utility" externalities are negative. Thus dU_A/dX_2^A will become more positive as A gives X_2 to B. A will give X_2 to B until he reaches point L' in Figure 2.12. At this point $dU_A/dX_2^A = 0$ and A is maximizing his utility given the constraint that the distribution of X_1 remains constant. Point L' is not a Pareto optimum since it is still possible for both individuals to gain from trade.

With a unilateral transfer the relevant portion of the contract curve will shrink from MP to MN. The portion KM is not efficient for A since it implies that, from point L, A will transfer some of both goods to B. At point L $dU_A/dX_1^A > 0$ and $dU_A/dX_2^A < 0$. A will be willing to transfer good X_2 since this will increase his utility but he will not be willing to transfer good X_1 since that would lower his utility. Even if his utility is increased when he transfers x_1 units of X_1 and x_2 units of X_2 to B, he can receive a greater increase in his utility if he transfers only x_2 units of X_2 to B and makes no transfers of X_1. Therefore the relevant portion of the contract curve when the initial distribution is at point L is the segment MP.

If individual A's initial distribution lies anywhere in the region CEF of Figure 2.12 then he can increase his utility by transferring units of X_2 to individual B until he has reached a point on the ridge line CE. If trade is not allowed this movement will maximize A's utility and increase B's utility. Points on CE are not Pareto efficient, except for point E, so the movement to the ridge line is only a movement toward Pareto optimality. The indifference curves of the two individuals are not tangent on the ridge lines, except at E, so points on the ridge line do not represent a social optimum. Pareto optimality has not been achieved since there is still the opportunity for exchanges which will make both individuals better off. If trade is allowed from points within CEF, then A will prefer to exchange X_2 for X_1 since this will increase his utility more than making a unilateral transfer. This is why we call the region CEF a region of possible charity. Region DEG is also a region of possible charity, but in this region X_1 is the good which yields external benefits to individual A. In this region A is willing to transfer units of X_1 to individual B until the ridge line DE is reached. He would prefer to exchange X_1 for X_2, however.

Note that if A is in a region of possible charity such as at point L in Figure 2.12 and if this is known to B, then B can use a strategy against A in order to increase his expected utility. If B realizes that A is in a region of possible charity he can refuse to make any exchanges until A has maximized his utility via transfers. After A makes the move from L to L' B can consent to exchange X_1 for X_2. By using this type of behavior B can insure that he will end up somewhere along the MN portion of the contract curve and not on the NP portion. All points on MN are superior to points on NP for individual B.

For this strategic behavior to work, A must realize that B may use it. This knowledge must be known by A when he formulates his utility function. If B attempts to use this strategic behavior when A does not expect it or is ignorant of it, then A may change his preferences in such a way that he becomes less benevolent towards B. The change in A's utility function would, of course, change A's indifference map, which would, in turn, affect the location of the contract curve and ridge lines. It may be possible that the shift in A's utility function will cause A to lose all interest in making a transfer to B. He certainly will desire to make a smaller transfer. Thus B's strategic behavior will not insure his reaching indifference curve I_B^2 if this strategic behavior causes a change in A's preferences. B's expected utility may in fact decrease rather than increase as a result of his strategic behavior.

Daly and Giertz [12, p. 3] state that the type of externality ("goods" or "utility") will have an effect upon whether A will desire to transfer goods or purchasing power to B. The authors state that "the existence of "goods" externalities causes marginal social and private rates of substitution to differ with the result that, in the presence of such externalities, trade will not lead to Pareto optimality" [12, p. 14]. They also state that "donors who are subject only to "utility" externalities will also prefer redistribution in the form of purchasing power" [12, p. 14]. Both of these statements contain elements of truth and falsehood. The implications of their statements are that when only "goods" externalities are present redistribution will be in the form of goods and the contract curve will not be reached and when only "utility" externalities are present redistribution will be in the form of purchasing power and the contract curve will be reached. These implications are not always realized as we shall see below.

In the first quotation the authors incorrectly used the word trade when they meant transfers. If our two individuals are allowed to engage in trade then there is no reason why they should not reach a point on the contract curve. If, however, only transfers are allowed, then, given that A is in a region of possible charity, he will make a transfer of goods to B so that he, A, has just reached his relevant ridge line. In this case, A's marginal rate of social substitution will not equal B's marginal rate of private substitution and Daly and Giertz are correct in stating that Pareto optimality will not be obtained.

The problem with their first statement lies deeper than this choice of words, however, because the truth of the statement hinges on their implicit assumption that A must not have a relevant, benevolent "goods" externality for every good in the consumption set. That is, the initial distribution must not be such that A is in his region of charity. If A is initially in his region of charity, then he will be able to reach his bliss point by transferring some of all goods to B. A's bliss point is on the contract curve which makes it a Pareto efficient point. Thus this Daly and Giertz argument holds only when the donor is in a region of possible charity.

In a two person world one cannot really make a transfer of purchasing power if all goods are in fixed supply. Purchasing power refers to any generalized medium of exchange that can be used to purchase goods or services. In the two person case one person can only receive what the other person gives up. If we equate a transfer of purchasing power in the two person case with a transfer of some of each good, then the existence of only "goods" externalities can lead to a transfer of "purchasing power" if the donor is originally in his region of charity.

The second Daly and Giertz statement concerns donors preferring redistribution in the form of purchasing power when only "utility" externalities are present. The rationale for transferring purchasing power is that the recipient is the best judge of what will maximize his utility. When the recipient maximizes his utility the donor's utility is supposedly maximized. In a two person, two good world in which the donor experiences only "utility" externalities the slope of the donor's utility curve is given by

$$\frac{dX_2^{\,A}}{dX_1^{\,A}} = - \frac{\dfrac{\partial U_A}{\partial X_1^{\,A}} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_1^{\,B}}}{\dfrac{\partial U_A}{\partial X_2^{\,A}} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_2^{\,B}}} \; . \tag{2.39}$$

The donor's (A's) desire to make transfers to the recipient (B) is influenced by the marginal benefits he receives from such transfers and by his marginal costs. It may be that B's marginal utility for good X_1 is larger than his marginal utility for good X_2. If given a choice he would consume good X_1 before he consumed good X_2. A might also hold X_1 more dear than X_2. If A's own marginal valuation of X_1 is great enough and that of X_2 small enough, then A will be willing to transfer good X_2 but not X_1 even though B's utility increases most rapidly when he consumes good X_1. A will be in a region of possible charity with respect to good X_2.

The existence of a "utility" externality only does not imply that the donor will be in a region of charity. Since he bases his transfer upon his costs as well as his benefits the donor may desire to transfer a specific good rather than general purchasing power (some of each good in the two person case). Moreover, the good the donor transfers may not be the one most desired by the recipient.

Let us jump ahead a bit and look briefly at the N-person case. If the recipient of a transfer in the N-person case is allowed to costlessly exchange the transferred good for other goods then a transfer of goods will yield the same result as a transfer of purchasing power. If he is not allowed to exchange a transferred good for other goods then the two types of transfers are not the same. The recipient will receive more utility from a transfer of purchasing power than from the transfer of an equivalent value of some good. The type of transfer will not be determined by the type of externality the donor faces, however. As we have seen in the two person case, both "goods" and "utility" externalities can lead to distributions in the donor's region of charity and region of possible charity.

The donor will make a transfer of purchasing power only when he is in his region of charity. When the recipient's purchases move the donor out of his region of charity, the donor will stop making transfers of purchasing power. If the recipient's purchases move the donor to a region of possible charity, the donor will be willing to transfer only the relevant good. If he is moved to the region of exchange or to his bliss point, the donor will stop making transfers altogether.

If both individuals have interdependent utility functions then the analysis is straightforward. In the two good case each individual may have a bliss point within the Edgeworth box. In Figure 2.13, A's bliss point is at point E_A and B's bliss point is at point E_B. The relevant portion of the contract curve is E_AE_B. The ridge lines for A (CE_AD and GE_AF) and for B (IE_BK and JE_BH) divide the Edgeworth box into nine regions numbered one through nine.[1]

[1]Boulding [4, pp. 70-71] presents a graph similar to Figure 2.13, but he incorrectly states the characteristics of all regions except those we have labeled 1, 2, and 3.

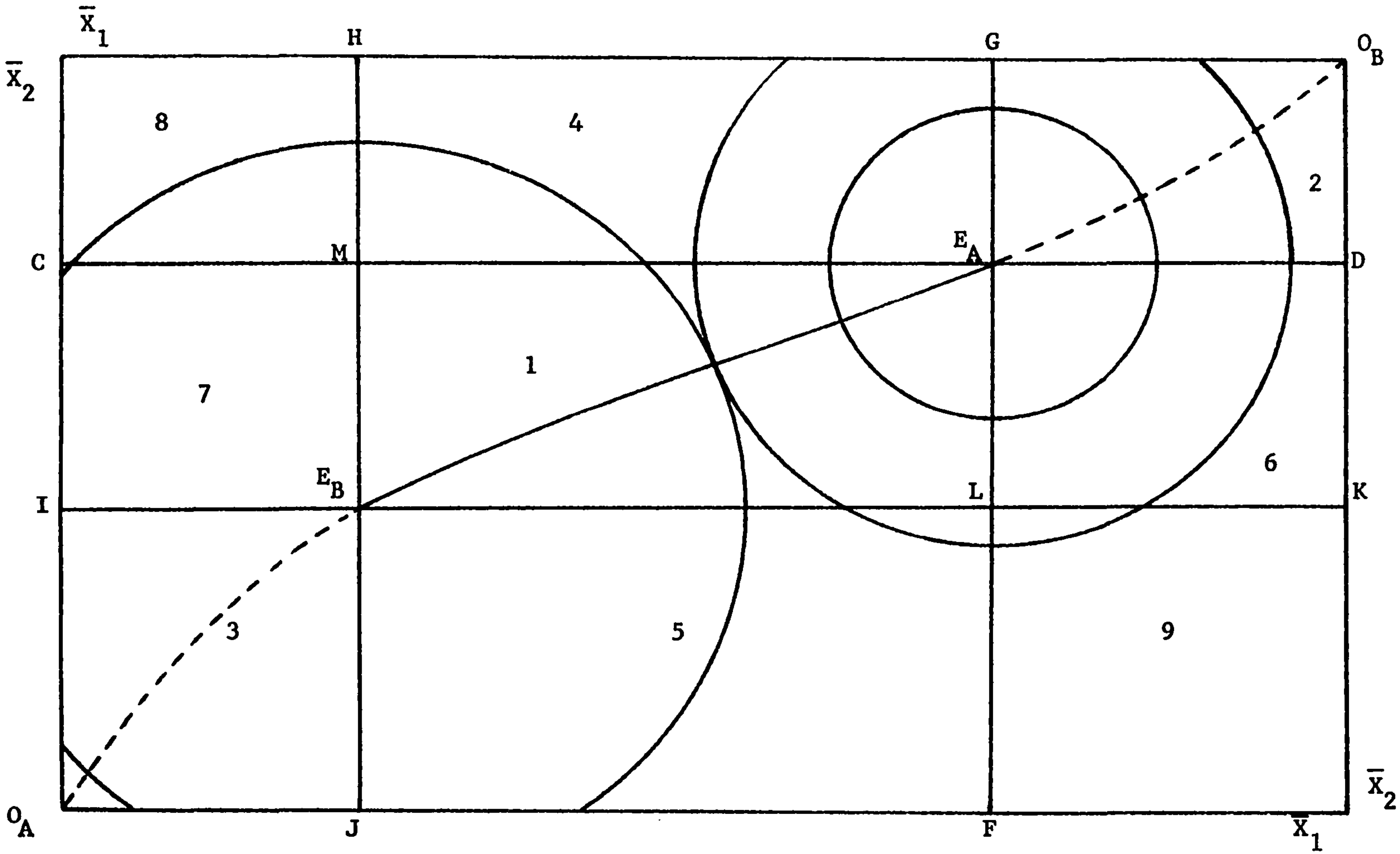

Figure 2.13. Edgeworth box when both consumers have interdependent utility functions

In region 1 both individuals exhibit the normal trade behavior discussed in the second section of this chapter. In region 2, A is charitable in both goods and he will transfer some of both goods to B so that the final distribution is at point E_A. In region 3, B is charitable in both goods and he will make a transfer to A so as to reach point E_B. In regions 8 and 9 each will be possibly charitable in one good, but not in the same good as the other individual. In region 8 A is possibly charitable in X_2 while B is possibly charitable in X_1. In region 9 each is possibly charitable in the other good. An initial distribution in region 8 will lead (without trade) to the point M. A will transfer X_2 to B and B will transfer X_1 to A. Since the point M lies at the intersection of two ridge lines this is the point that will be reached through transfers alone. If trade is allowed, the final distribution will be on the E_BE_A portion of the contract curve. Any initial allocation in region 9 will lead either to a point on E_BE_A if trade is allowed or to point L if trade is not allowed.

In regions 4, 5, 6 and 7 only one individual will be possibly charitable and then only in one good. In region 4 A is possibly charitable in X_2 and in region 6 he is possibly charitable in X_1. B is possibly charitable in X_1 and X_2 in regions 7 and 5 respectively.

Another possibility when both individuals have <u>very</u> strong benevolent externalities is for the bliss points E_A and E_B to reverse position as in Figure 2.14. In Figure 2.14 the ridge lines for A are CE_ALD and FE_AMG and those for B are IME_BK and JLE_BH. In this figure there is no contract curve as such. Region 1 is, in effect, a contract region

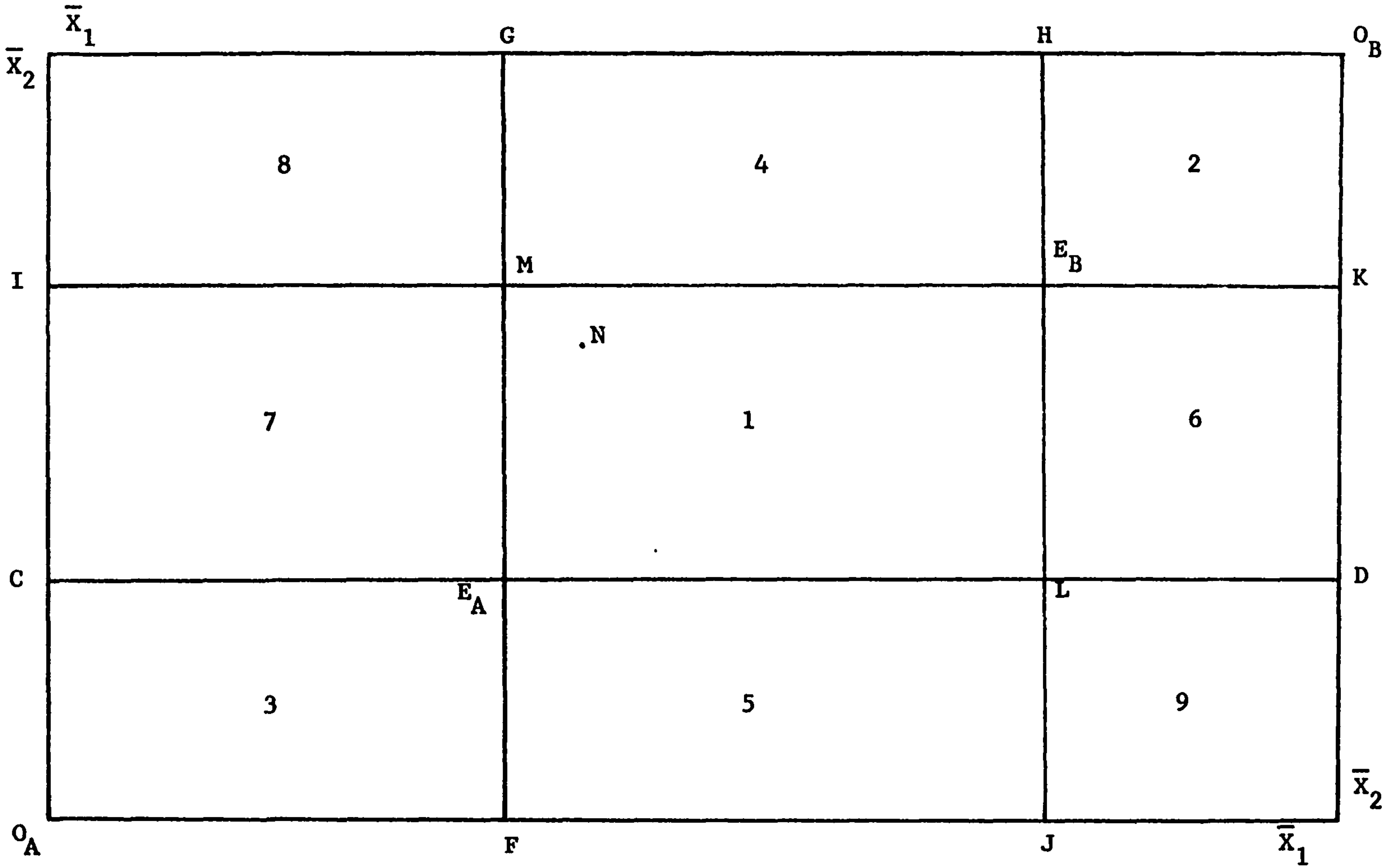

Figure 2.14. The Edgeworth box when both individuals have very strong benevolent feelings

since neither individual can make a transfer to the other without making the other worse off. If the initial allocation of goods leads to a distribution at point N in Figure 2.14, then A will desire to transfer some of both goods to individual B in such proportion that the final distribution is at point E_A. Likewise B will desire to move from N to E_B via transfers of both goods to A. Each will desire to transfer some of both goods to the other and, as long as each has the option of not accepting the other's gift, the original allocation N will be the final allocation. If it is considered improper to refuse to accept a gift, then each will accept the gift which moves the donor to his bliss point. The recipient will, however, either immediately or after a respectable period of time offer a gift to the other which will move the current donor to his bliss point. The final solution is not a stable solution if our individuals do not have the option of refusing a gift.

If the initial distribution is in region 2 or 3 then there will be an unambiguous movement via transfers to E_B or E_A respectively. Initial distributions in regions 8 or 9 will lead via trade and/or transfer to M or L respectively. In each of the other four regions (4, 5, 6, and 7) one individual will desire to transfer some of each good and the other individual will desire to transfer some of one good. For example, in region 4 in Figure 2.14 individual A desires to transfer both X_1 and X_2 to individual B while B desires to transfer X_1 to A. Assuming that each can refuse to accept the other's offer, no X_1 will change hands but A will transfer some X_2 to individual B - not to a point on E_AL, A's ridge line, but to a point on ME_B, B's ridge line. Thus if the initial distribution is in regions 4, 5, 6 or 7 there will be a movement via transfer from that distribution to the nearest ridge line bordering region 1. The "wrong" individual will maximize his utility given the constraints but the other individual will increase his utility by making the transfer.

The results of this section do generalize easily to the n-good case. If "utility" externalities are present then there is always the possibility that a region of charity will occur. We say there is the possibility it may occur because the benevolent feelings of both individuals may be so weak that the bliss points of both individuals lie outside the hypercube that forms the commodity set of the individuals. If this region does occur, then once an individual reaches this region he will transfer some of every good to the other individual until his bliss point is reached. If an individual is in a region of possible charity then he will be willing to transfer at least one but less than n-goods to the other individual. In the n-good case it is possible for up to n-1 goods to be in an individual's region of possible charity. The individual will be willing to transfer the goods in his region of possible charity until he reaches a ridge hyperplane or until the other individual refuses to accept the good(s). When the individual reaches a ridge hyperplane he will stop transferring one or more goods but will continue to transfer others until he reaches the boundary of either the pure exchange set or the pure contract set. In the two good case these sets are region 1 of Figure 2.13 and Figure 2.14 respectively.

If only "goods" externalities are present in the n-good case then there will be no region of charity unless one or both individuals have benevolent externalities from all n goods. If externalities are only present on $p < n$ goods then the individual may have regions of possible charity but he would not have a region of charity. The analysis of these regions is exactly like that discussed under "utility" externalities above.

An Application of Interdependent Utility Functions[1]

In this section the analysis developed in the last section will be applied to explain the increase in charitable contributions that are often observed following natural or personal disasters. De Alessi [14] notes that charity tends to increase immediately following a disaster and he attempts to explain this increase in giving through an interdependent model. His model assumes that charity is one of many goods in the consumer's consumption set and that in equilibrium the consumer will equate $MU_{Charity}/P_{Charity}$ with MU_i/P_i for all other goods. After a disaster a dollar's worth of charity will yield more utility to the donor than it did before, not because the donor's marginal utility of charity has increased due to a change in his preferences but because his knowledge of the disaster has reduced his search costs for a suitable object of charity.

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[1]See [13, p. 23].

Our approach will differ from De Alessi's so that we may use the tools developed in the preceding section. Since we are restricting ourselves to a two person, two good world we will define a charitable action as any transfer of goods from one individual to the other. For simplicity we will assume that B has a strictly private utility function and that A's utility function exhibits benevolent externalities with regards to B. We assume that the amounts of the two goods (X_1 and X_2) are fixed at $\bar{X}_1$ units and $\bar{X}_2$ units. Let us assume that in the pre-disaster state the individuals have reached an equilibrium at point L in Figure 2.15. Point E is, of course, A's bliss point and the contract curve is 0_ALE. $I_A{}^1$ and $I_B{}^1$ are the indifference curves for A and B respectively that pass through the point L. At this equilibrium point B is consuming $x_1{}^B = \bar{X}_1 - x_1{}^A$ units of good X_1 and $x_2{}^B = \bar{X}_2 - x_2{}^A$ units of good X_2.

Now let some disaster strike individual B. Suppose that for some reason he loses one half of his holdings of good X_1. His holding of X_2 we assumed to be unaffected by the disaster. B will now have holdings of $x_2{}^B$ units of X_2 as before and $x_1{}^{B^1} = 1/2\ x_1{}^B$ units of X_1. In Figure 2.15, B's origin will shift from 0_B to $0_B{}^1$. The total amount of X_1 available to the two individuals will shrink from $\bar{X}_1$ to $\bar{\bar{X}}_1 = \bar{X}_1 - 1/2\ x_1{}^B$. Point L still indicates the distribution of goods between the two individuals but it no longer represents an optimal distribution. The slopes of both indifference curves through L will change and these new indifference curves will represent lower levels of utility for each individual.

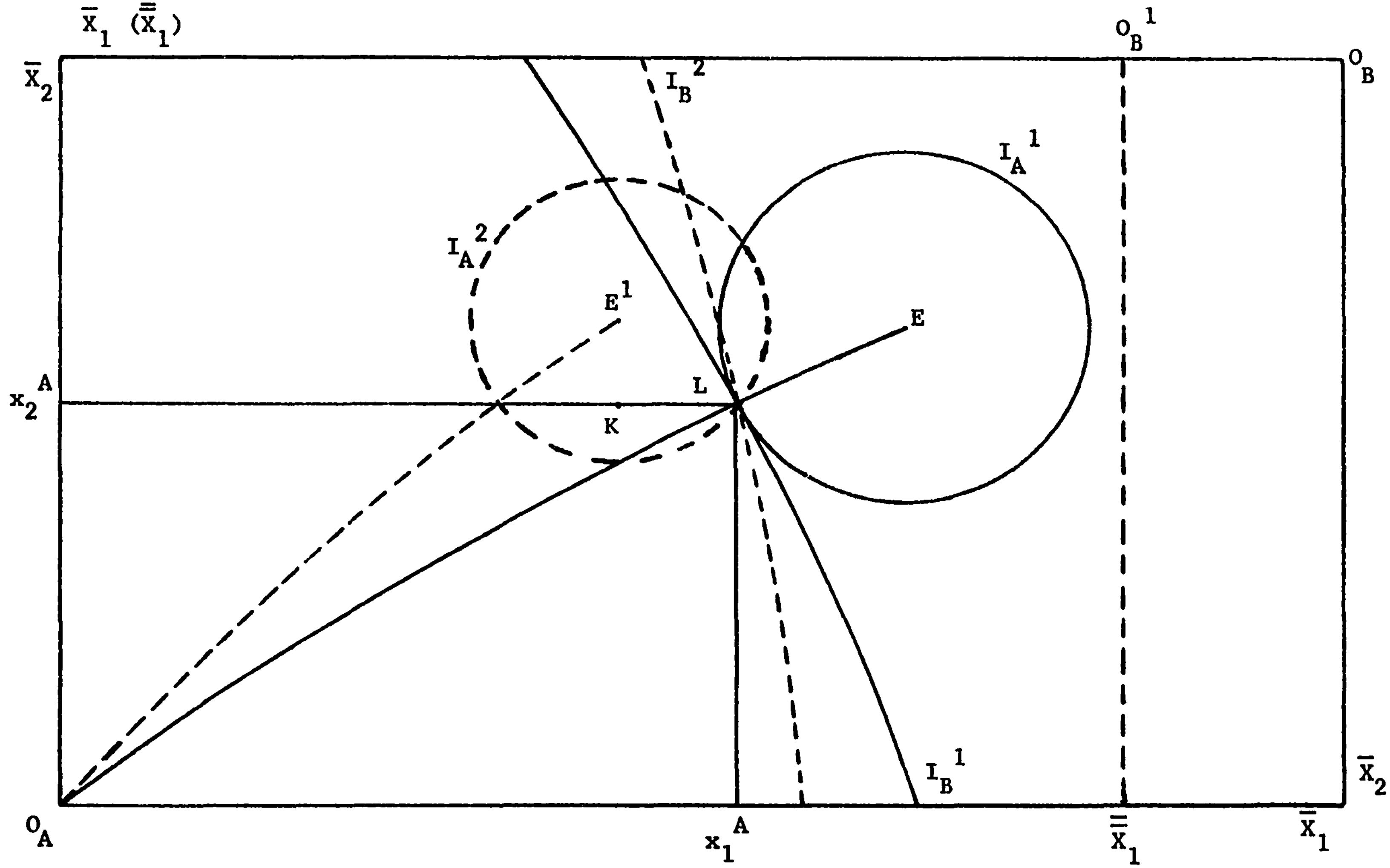

Figure 2.15. The effects of disaster

The slope of B's new indifference curve through point L ($I_B^{\ 2}$) will be steeper than his original curve through that point ($I_B^{\ 1}$). To see this let us look at the total derivative of B's utility function:

$$dU_B = \frac{\partial U_B}{\partial X_1^{\ B}}\, dX_1^{\ B} + \frac{\partial U_B}{\partial X_2^{\ B}}\, dX_2^{\ B}. \tag{2.40}$$

B's holdings of X_2 have not changed so $dX_2^{\ B} = 0$. His holdings of X_1 have decreased and since $\partial U_B/\partial X_1^{\ B} > 0$ B's total utility will fall. As it falls $\partial U_B/\partial X_1^{\ B}$ will increase since $\partial^2 U_B/\partial X_1^{\ B^2} < 0$. At the new, lower level of utility the equation for B's indifference curve will be

$$\frac{dX_2^{\ B}}{dX_1^{\ B}} = - \frac{\dfrac{\partial U_B}{\partial X_1^{\ B}}}{\dfrac{\partial U_B}{\partial X_2^{\ B}}} \quad . \tag{2.41}$$

This is the same equation he has for all his indifference curves including $I_B^{\ 1}$. Since B's consumption of X_2 is constant, $\partial U_B/\partial X_2^{\ B}$ is constant both before and after the disaster. The disaster decreased his holdings of X_1 which increased $\partial U_B/\partial X_1^{\ B}$ so $dX_2^{\ B}/dX_1^{\ B}$ after the disaster is greater than $dX_2^{\ B}/dX_1^{\ B}$ before the disaster and the slope of the new indifference curve through point L is steeper than the old.

A's new indifference curve through the point L will become less negatively sloped and may even become positively sloped. This new indifference curve will also represent a lower level of utility for A. The total derivative for A's utility function is

$$dU_A = \frac{\partial U_A}{\partial X_1^A} dX_1^A + [\frac{\partial U_A}{\partial X_1^B} + \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_1^B}] dX_1^B + \frac{\partial U_A}{\partial X_2^A} dX_2^A$$

$$+ [\frac{\partial U_A}{\partial X_2^B} + \frac{\partial U_A}{\partial U_B}\frac{\partial U_B}{\partial X_2^B}] dX_2^B. \tag{2.42}$$

When disaster strikes individual B, A's utility is lowered if $\partial U_A/\partial X_1^B$ and $\partial U_A/\partial U_B$ are positive as we have assumed. The decrease in B's holdings of X_1 will increase $\partial U_A/\partial X_1^B$, $\partial U_A/\partial U_B$ and $\partial U_B/\partial X_1^B$ if all second derivatives ($\partial^2 U_A/\partial X_1^{B^2}$, $\partial^2 U_A/\partial U_B^2$ and $\partial^2 U_B/\partial X_1^{B^2}$) are negative as assumed. After the disaster U_A at point L has been lowered even though A's holdings of X_1 and X_2 have not changed. The equation for A's indifference curve is

$$\frac{dX_2^A}{dX_1^A} = - \frac{\dfrac{\partial U_A}{\partial X_1^A} - \dfrac{\partial U_A}{\partial X_1^B} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_1^B}}{\dfrac{\partial U_A}{\partial X_2^A} - \dfrac{\partial U_A}{\partial X_2^B} - \dfrac{\partial U_A}{\partial U_B}\dfrac{\partial U_B}{\partial X_2^B}} \tag{2.43}$$

given $dX_1^A = - dX_1^B$. This last equation will not hold at the time of disaster but it will hold before and after the disaster. The denominator of the right hand side will be slightly smaller in the post-disaster period since $\partial U_A/\partial U_B$ will have increased. The numerator will be even smaller since $\partial U_A/\partial X_1^B$, $\partial U_A/\partial U_B$, and $\partial U_B/\partial X_1^B$ will all have increased. Thus A's indifference curve through the point L will be less negatively sloped in the post-disaster period and may even have a positive slope. Individual A may move into a region of possible charity after the disaster.

If the increase in $\partial U_A/\partial U_B$ is large enough it is possible that A will move into his region of charity. We would expect A to move into his region of charity more often after a disaster that reduced B's holdings of all goods than after a disaster that reduced B's holdings of only a few goods. Whether A is moved to a region of possible charity or to his region of charity will depend in part upon the degree of his benevolence toward B and in part upon the amount of devastation the disaster levies upon B.

In Figure 2.15 the post-disaster situation is depicted by the dotted lines. B's new origin has moved to $O_B{}^1$ from O_B and his new indifference curve through L is $I_B{}^2$. A's new indifference curve through L is the dotted circle $I_A{}^2$. Both $I_A{}^2$ and $I_B{}^2$ represent lower levels of utility than $I_A{}^1$ and $I_B{}^1$. A's bliss point has moved from E to E'. The new contract curve (OE') will be above and to the left of the old contract curve. This is as expected since B's loss of X_1 makes X_1 more dear to him and makes him more willing to exchange X_2 for X_1. A is shown to be in a region of possible charity and he is willing to transfer LK units of X_1 to B although he would rather exchange his X_1 for X_2 at terms that would allow him to move from L to E'.

This application of interdependent utility functions has shown that the observed charitable behavior which follows disaster need not be brought about by a change in the tastes of the philanthropic individual but can be explained as rational and predictable behavior on the part of this individual.

CHAPTER III. THE N PERSON CASE

In Chapter II we analyzed individual voluntary transfer behavior in a world in which there were only two individuals. While convenient for expressing basic concepts and for ease of graphical exposition the two-person case is not an accurate description of reality when there are more than two individuals in society. In this chapter we shall extend the analysis of the two-person case to a general N-person case. We shall continue to utilize the interdependent utility function as the prime generator for interpersonal transfers; however, as we shall see, the analysis becomes more complicated in a many person world.

We will work through the N-person case in a step-by-step fashion. First we shall look at the case where only one individual has an interdependent utility function. In this world there will be N-1 possible recipients of transfers but only one possible donor. Next we will look at a case where some subset of society, say K individuals, have interdependent utility functions. Initially we will assume that the recipient individuals of each donor form a set that is mutually exclusive of the recipients of all other donors. We will remove this assumption and move to situations involving common recipients for several donors and donors being recipients of other donors. Finally we shall examine the situation in which all recipients as well as donors have interdependent utility functions. In the last section we shall briefly discuss the formation of charitable organizations.

The Single Donor

In this section and in most of this chapter we will make the following assumptions. We assume that there are N individuals in the society of which K are donors ($1 \le K \le N$) and at least N-K are potential recipients. K = 1 in this section. There are M goods (X_1, ..., X_m) each of which is initially distributed among the N individuals and each of which is fixed in supply during each period of time. As in Chapter II, we are not concerned here with how the M goods are produced nor are we concerned with how they are initially distributed among the N members of society. We are concerned with how they are redistributed after the initial distribution has taken place.

It is assumed that each consumer acts rationally. He behaves in such a manner that he maximizes his own utility. It is assumed that all transfers can be made costlessly. The assumption of nonappropriation is also made. Individuals are free to make voluntary transfers but they are not free to seize the goods which belong to another.

The utility function for the donor individual is as follows:

$$U_1 = U_1(X_{11}, X_{12}, \ldots, X_{1M}; X_{21}, \ldots, X_{2M}; \ldots; X_{N1}, \ldots, X_{NM}; U_2, \ldots, U_N). \tag{3.1}$$

Individual 1 is concerned not only about his own level of consumption but also about the consumption and utility levels of the other N-1 individuals in his society.

The utility function of the other N-1 individuals is given by:

$$U_i = U_i(X_{i1}, X_{i2}, \ldots, X_{iM}); \quad i = 2, 3, \ldots, N. \tag{3.2}$$

These individuals are selfish in the usual economic connotation of that word.

It is assumed that these utility functions are twice differentiable and that the functions and their derivatives are continuous. The utility maximizing conditions for the N-1 potential recipients is found by taking the total derivative of their utility functions and setting it equal to zero:

$$dU_i = \frac{\partial U_i}{\partial X_{i1}} dX_{i1} + \frac{\partial U_i}{\partial X_{i2}} dX_{i2} + \ldots + \frac{\partial U_i}{\partial X_{iM}} dX_{iM} = 0. \qquad (3.3)$$

The first order conditions for a utility maximum are that the slope of the indifference surface in the plane of any two goods must equal the negative of the ratio of the marginal utilities for those two goods. The consumption of all other goods is assumed to be constant.

$$\frac{dX_{ij}}{\partial X_{iK}} = - \frac{\partial U_i/\partial X_{iK}}{\partial U_i/\partial X_{ij}}\,; \quad dX_{iL} = 0, \quad L \neq j, K. \qquad (3.4)$$

The set of second partial derivatives is the bordered Hessian which is assumed to be negative definite so that the first order conditions lead to a utility maximum.

The total differential of the donor's utility function is:

$$dU_1 = \sum_{j=1}^{m} [\frac{\partial U_1}{\partial X_{ij}} dX_{ij} + \sum_{i=2}^{n} [\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i} \frac{\partial U_i}{\partial X_{ij}}] dX_{ij}]. \qquad (3.5)$$

Assuming again that the bordered Hessian is negative definite, the first order conditions for a utility maximum for the donor are that:

$$\frac{dX_{1K}}{dX_{1L}} = - \frac{\frac{\partial U_1}{\partial X_{1L}} + \sum_{i=2}^{n} [\frac{\partial U_1}{\partial X_{iL}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{iL}}] \frac{dX_{iL}}{dX_{1L}}}{\frac{\partial U_1}{\partial X_{1K}} + \sum_{i=2}^{n} [\frac{\partial U_1}{\partial X_{iK}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{iK}}] \frac{dX_{iK}}{dX_{1K}}} \qquad (3.6)$$

with $dX_{1j} = 0$ for $j \neq K, L$.

The first term in both the numerator and denominator of the right hand side is the donor's own consumption component of the ratio of marginal utilities. If he were not concerned with the welfare or consumption levels of other individuals, then these would be the only terms on the right hand side. He would have the same first order conditions as the other economically selfish individuals.

The second term in the numerator and denominator of the right hand side measures the "goods" and "utility" externalities experienced by the donor individual. The first portion of this second term tells how the donor's utility will directly react to an increase in the recipients' consumption of a particular good. We will assume that $\partial U_1/\partial X_{ij} \gtreqless 0$ and $\partial^2 U_1/\partial X_{ij}^{\,2} \leq 0$. There is no reason why $\partial U_1/\partial X_{ij}$ should not become negative, if it is initially positive, after X_{ij} reaches some level that is critical in either an absolute or a relative sense. We assume that it is positive if the donor makes voluntary transfers which are brought about by benevolent feelings toward the potential recipient.

The second portion of this second term, $\partial U_1/\partial U_i \cdot \partial U_i/\partial X_{ij}$, tells how the donor's utility increases with increases in the recipient's utility. Note that in this case $\partial U_i/\partial X_{ij}$ is not individual I's marginal utility of good j but rather is the donor's inference of that marginal utility. The dX_{ij}/dX_{1j} term is the negative of the fraction of the donor's transfer of the j^{th} good going to the i^{th} recipient. Recall that

$$X_{1j} + X_{2j} + \ldots + X_{nj} = \bar{X}_j \tag{3.7}$$

so that

$$d\bar{X}_j = 0 = dX_{1j} + dX_{2j} + \ldots + dX_{nj} \tag{3.8}$$

which implies that

$$dX_{1j} = - \sum_{k=2}^{n} dX_{kj} \ . \tag{3.9}$$

Therefore,

$$\frac{dX_{ij}}{dX_{1j}} = - \frac{dX_{ij}}{\sum\limits_{k=2}^{n} dX_{kj}} \ . \tag{3.10}$$

In the two person case this fraction is always minus one since the recipient necessarily receives what the donor transfers.

Given equation 3.6, when will the donor have an incentive to make a voluntary transfer to one or more of the potential recipients? The answer is, of course, when he is in a region of possible charity or the region of charity. In the two person case the donor was in a region of possible charity whenever

$$\frac{\partial U_1}{\partial X_{1j}} - [\frac{\partial U_1}{\partial X_{2j}} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_{2j}}] < 0 \quad \text{for some } j, \tag{3.11}$$

and he was in the region of charity whenever this condition held for all j. Of course, $dX_{2j}/dX_{1j} = -1$ in the two person case. In the many person case the analogous condition might seem to be:

$$\frac{\partial U_1}{\partial X_{1j}} - \sum_{i=2}^{n} \left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right] < 0 \quad \text{for some } j, \tag{3.12}$$

but this is not necessarily the case. This condition is neither sufficient nor necessary for the donor to voluntarily transfer good j to one or more recipients if the good j is a strictly private good. If good j is a private good when held by the donor but exhibits public goods attributes when transferred, then, under certain circumstances, equation 3.12 is the necessary condition which must be satisfied before a transfer will be made. The certain circumstances will be given later in this section. An example of a good which is a purely private good when held by the donor and a public good after it has been transferred would be works of art given by a private collector to a public museum.

Since the good being transferred is not generally a public good in the sense that every recipient can consume the total amount of the good without reducing the amount available to other recipients, it follows that if the donor transfers one unit of good j then this unit of good j must be divided among the other N-1 individuals. (Some individuals can receive a zero share of the transfer.) In order for the transfer of a private good to increase the donor's utility it is necessary that

$$\frac{\partial U_1}{\partial X_{1j}} - \left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right] < 0 \tag{3.13}$$

for at least one of the other N-1 individuals. If this is the case then the donor may make transfers (of good j) to individual I until:

$$\frac{\partial U_1}{\partial X_{1j}} - \left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right] = 0, \tag{3.14}$$

or until

$$\frac{\partial U_1}{\partial X_{1j}} - \left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right] < 0 \tag{3.15}$$

<u>and</u>

$$\left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right] = \left[\frac{\partial U_1}{\partial X_{kj}} + \frac{\partial U_1}{\partial U_k}\frac{\partial U_k}{\partial X_{kj}}\right]. \tag{3.16}$$

If this latter event should occur then the donor might make transfers to both individuals I and K in such proportions that the above equality is maintained. This will continue until

$$\frac{\partial U_1}{\partial X_{1j}} - \sum_{i=2}^{P}\left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right]\frac{dX_{ij}}{\sum\limits_{i=2}^{P} dX_{ij}} = 0, \tag{3.17}$$

where P is equal to the number of individuals receiving transfers from the donor plus one.

An example will clarify this point. Suppose we have one donor, individual 1, and three potential recipients, individuals 2, 3, and 4. The initial distribution of goods is assumed given. With this initial distribution suppose that individual 1's marginal utility with respect to some good j is given by

$$.4\ dX_{1j} + .6\ dX_{2j} + .3\ dX_{3j} + .7\ dX_{4j}. \tag{3.18}$$

In this case $\partial U_1/\partial X_{1j} = 0.4$; $\partial U_1/\partial X_{2j} + \partial U_1/\partial U_2 \cdot \partial U_2/\partial X_{2j} = 0.6$; $\partial U_1/\partial X_{3j} + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_{3j} = 0.3$; and, $\partial U_1/\partial X_{4j} + \partial U_1/\partial U_4 \cdot \partial U_4/\partial X_{4j} = 0.7$. It is readily apparent that individual 1 can increase his utility by transferring some units of good j to either of individuals 2 or 4. He could not increase his utility by transferring to individual 3 since he would gain 0.3 utils per unit transferred but would give up 0.4. Transferring one unit of good j to individual 2 would yield individual 1 a net utility increase of 0.2 utils. A similar transfer to individual 4 would give him a net increase of 0.3 utils. Therefore he will make initial transfers to individual 4 only, since this will give him the greatest increase in utility.

As individual 1 transfers units of good j to individual 4 his own marginal valuation of the good will increase since he has fewer units of the good to consume. Since individual 4 is now consuming more of good j individual 1's marginal valuation of individual 4's consumption will fall. If the marginal valuation of his own consumption rises faster than his marginal valuation of individual 4's consumption falls then individual 1 might make transfers to individual 4 only. His post-transfer marginal utility function might look like

$$0.62\ dX_{1j} + 0.6\ dX_{2j} + 0.3\ dX_{3j} + 0.62\ dX_{4j}. \tag{3.19}$$

Since he made no transfer to individuals 2 and 3, $dX_{2j} = dX_{3j} = 0$. All transfers went to individual 4 so that $dX_{1j} = -dX_{4j}$ and by substitution

$$0.62\ dX_{1j} - 0.62\ dX_{1j} = 0. \tag{3.20}$$

Equation 3.14 is satisfied.

If individual 1's marginal valuation of his own consumption rises more slowly than the fall in his marginal valuation of individual 4's consumption, then he will reach a point where he receives as much additional utility by transferring some units of good j to individual 2 as he does by transferring them to individual 4. Suppose after transferring some units of good j to individual 4 the donor's marginal utility function (with respect to good j) looks as follows:

$$0.48\ dX_{1j} + 0.6\ dX_{2j} + 0.3\ dX_{3j} + 0.6\ dX_{4j}. \qquad (3.21)$$

At this point individual 1 will divide his transfers between individuals 2 and 4. He will divide the transfers in such a way that his marginal valuation for the consumption of the two recipients remains the same. This does not mean that he will divide his transfers equally between the two individuals. If the marginal valuation for one recipient falls more rapidly than that for the other recipient then the first recipient will receive a smaller share of the transfers than the second recipient.

Individual 1 will make transfers until the marginal valuation of his own consumption is equal to his marginal valuation of the consumption of each individual recipient. At this point individual 1's marginal utility function might look like

$$0.58\ dX_{1j} + 0.58\ dX_{2j} + 0.58\ dX_{4j} + 0.3\ dX_{3j}. \qquad (3.22)$$

Dividing through by dX_{1j} and assuming that at this point $dX_{2j} = -2/3\ dX_{1j}$ and $dX_{4j} = -1/3\ dX_{1j}$ we find:

$$0.58 - 0.58\ (2/3) - 0.58\ (1/3) + 0.3\ (0) = 0. \qquad (3.23)$$

Thus equation 3.17 is satisfied.

Before this example we stated that the donor <u>might</u> make transfers if favorable externalities existed, not that he <u>would</u> make transfers. We know from Chapter II that he would definitely make transfers only if he were in his region of charity. If he were in his region of possible charity, the donor would prefer to trade the externality generating goods for other goods at a price below the market price only if his trading partner(s) were the individual(s) causing the "goods" or "utility" externality. Even though the donor would prefer to make his transfer by trading at less than the market terms of trade he would still be able to increase his utility by freely transferring the good to the chosen recipient(s).

It should be noted that as in Chapter II, if the donor does maximize his utility by making free transfers to one or more recipients, the resulting post-transfer position will not be a Pareto optimal position. At this point either the numerator or denominator of equation 3.6 will be zero so that the slope of the donor's indifference surface will be either zero or infinity in the relevant plane. The slope of the recipient's indifference curve in that same plane will be negative. The transfer does not lead to a Pareto efficient point.

Observe that in the case where the good in question is a purely private good the donor could have malevolent feelings toward all individuals in the society save one and still transfer some units of that good. If his benevolent feelings toward that one individual, say individual K, were such that

$$\frac{\partial U_1}{\partial X_{1j}} - \left[\frac{\partial U_1}{\partial X_{kj}} + \frac{\partial U_1}{\partial U_k}\frac{\partial U_k}{\partial X_{kj}}\right] < 0 \tag{3.24}$$

even though

$$\sum_{i=2}^{N} \left[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right] < 0, \tag{3.25}$$

he could still increase his utility by making a transfer of good j to individual K.

A few points should be noted in regard to the transfer of a strictly private good. First, it is not necessary for the first transfer to be made to the poorest or neediest person in the society. If the donor makes a transfer, he will make the transfer to that individual whose needs and welfare most concern him. That is, he will make the initial transfer to the individual causing the largest externality $[\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}]$, and this individual will not necessarily be the poorest individual in the society. The question of need will no doubt be a factor influencing the donor's utility function, but there is no reason why this factor should carry the greatest weight in the determination of that function. We shall shortly clarify this point with an example from the literature.

A second point to be noted is that even if the donor realizes that the recipient is not particularly fond of the transferred good ($\partial U_i/\partial X_{ij}$ is small) and that he gets little utility from the consumption of additional units of the good, the donor may still transfer that good to the recipient if: 1) the donor looks upon the good as a merit good whose consumption is good for the recipient ($\partial U_1/\partial X_{ij}$ is large); or, 2) if the donor incurs little loss in utility from not consuming the good himself ($\partial U_1/\partial X_{1j}$ is small compared with $\partial U_1/\partial X_{1k}$ for all $k \neq j$). Note that if the potential recipient does not desire the good ($\partial U_i/\partial X_{ij} < 0$ from individual I's point of view) then we assume that he can refuse the transfer. The fact that the donor offers him a good or goods does not mean he is obliged to accept them.

A third point to note is that even if the donor does not really think the recipient should consume a particular good ($\partial U_1/\partial X_{ij}$ is zero or negative) he may still make the transfer of that good if: 1) he knows it will increase the recipient's utility by a large amount ($\partial U_i/\partial X_{ij}$ is large and $\partial U_1/\partial U_i \cdot \partial U_i/\partial X_{ij} > \partial U_1/\partial X_{1j} - \partial U_1/\partial X_{ij}$); or 2) he receives little utility from the good himself. In this latter case it will still be necessary that $\partial U_1/\partial U_i \cdot \partial U_i/\partial X_{ij} > \partial U_1/\partial X_{1j} - \partial U_1/\partial X_{ij}$.

In regards to the point concerning making transfers to the neediest individual first, we should like to refer the reader to the articles by Hochman and Rodgers [22] and Von Furstenberg and Mueller [49]. Hochman and Rodgers hypothesized a situation in which every person made a transfer of money to each person whose initial income was lower than his own and received a money transfer from each individual whose initial income was higher than his own. The size of these transfers was based on the size of the initial income differential between income classes. They did not propose a specific model which would bring this about.

Von Furstenberg and Mueller on the other hand presented a model which had transfers being made only to individuals in the lowest income class. We will call the Von Furstenberg and Mueller approach the income floor approach to transfers. Brennan and Walsh [8] stated that they considered the Hochman and Rodgers formulation to be the more general but that they were unable to obtain the Von Furstenberg and Mueller results through any meaningful modification of the Hochman and Rodgers formulation.

If the level of transfers is based solely upon initial income differentials, regardless of the level of incomes as in the Hochman and Rodgers formulation, then an income floor result cannot be obtained unless there are only two income groups. If there are three income groups, then the Hochman and Rodgers formulation can lead to the appearance of an income floor result if the amount the richer group transfers to the middle income group is just equal to the amount the middle income group transfers to the lower income group. Under the Hochman and Rodgers assumption, this will occur whenever the income differential between the higher and middle groups is just equal to the differential between the middle and lower group. With more than three income groups we will never observe an income floor under the Hochman and Rodgers formulation. The Von Furstenberg and Mueller income floor approach will never lead to a situation where transfers are made to a group other than the group with the lowest income. Neither model is the more general. Both are special cases of a more general model that will be developed below.

Before proceeding we should note that Hochman and Rodgers and Von Furstenberg and Mueller assumed that all transfers were processed through a collective agency. Individuals were taxed according to their marginal benefit from transfers and the transfers were allocated in accordance with the taxpayers' wishes. We shall assume that all transfers are voluntary and that there is only one donor. We are assuming the existence of only one potential donor to be consistent with the objectives of this section. In our analysis we shall follow the other authors in assuming that only income levels enter an individual's utility function. We realize that this assumption is restrictive, that utility interdependence may flow from wealth, from the consumption of certain goods, from particular forms of behavior, or from any of a number of other non-income variables. We use income levels as utility function arguments in order to simplify the analysis.

Both sets of authors started from a similar set of assumptions in developing their models. They both assumed an interpersonal utility function with income levels as arguments:

$$U_i = f_i(Y_1, Y_2, \ldots, Y_i, \ldots, Y_n).^1 \tag{3.26}$$

They also assumed that 1) all transfers must lead to a Pareto better position; 2) all transfers flow from persons with higher incomes to persons with lower incomes; 3) the initial income ordering cannot be reversed, although two income levels may merge; and, 4) everyone faces the same set of prices for goods and services and the same interest (discount) rate. They assumed nonsatiation with respect to own income:

$$\frac{\partial U_i}{\partial Y_i} > 0; \quad i = 1, 2, \ldots, n. \tag{3.27}$$

For the donor to receive utility from a transfer in light of the preceding, it must follow that

$$\frac{\partial U_i}{\partial Y_j} > \frac{\partial U_i}{\partial Y_i} > 0; \quad \text{for some } j,^2 \quad dY_j = -dY_i, \tag{3.28}$$

[1] Strictly speaking, the authors assumed that U_i held as arguments only those Y_j such that $Y_j \le Y_i$.

[2] $\partial U_i/\partial Y_i$ corresponds to $\partial U_1/\partial X_{1j}$ in the terminology we developed while $\partial U_i/\partial Y_j$ corresponds to $[\partial U_1/\partial X_{ij} + \partial U_1/\partial U_i \cdot \partial U_i/\partial X_{ij}]$.

where j refers to the recipient of the transfer and i to the donor.

In the case where all individuals have interdependent utility functions the criterion for the recipient to be willing to accept the transfer (we assume that he always has the option of refusal) is:

$$\frac{\partial U_i}{\partial Y_j} > \frac{\partial U_i}{\partial Y_i}; \quad dY_j = -dY_i. \tag{3.29}$$

Given equations 3.27, 3.28 and 3.29 and the assumption of consumer sovereignty, the decision as to whether any transfer will be made rests with the potential donor. In the case where all recipients have purely private utility functions, the transfer would always be accepted.

The Von Furstenberg and Mueller specification of the utility function was specifically chosen so that it would yield the floor level of income results. Their utility function was of the Cobb-Douglas form:

$$U_i' = Y_i^a \prod_{j=i}^{i-1} \left(\frac{Y_j}{Y_i}\right)^{b'} ; \quad i = 1, \ldots, n, \tag{3.30}$$

or

$$U_i = \log U_i' = (a-(i-1)b') \log Y_i + b' \sum_{j=1}^{i-1} \log Y_j, \tag{3.31}$$

where individual 1 had the lowest income and individual N the highest. If we set the total derivative of this function equal to zero and assume that the appropriate second order conditions hold, then for individual I to maximize his utility the following must hold:

$$\frac{b'}{Y_1} dY_1 + \frac{b'}{Y_2} dY_2 + \ldots + \frac{b'}{Y_{i-1}} dY_{i-1} \leq \frac{-(a-(i-1)b')}{Y_i} dY_i. \tag{3.32}$$

Generally we would assume that

$$\frac{b'}{Y_1} dY_1 + \frac{b'}{Y_2} dY_2 + \ldots + \frac{b'}{Y_{i-1}} dY_{i-1} = \frac{-(a-(i-1)b')}{Y_i} dY_i \qquad (3.33)$$

for individual I to maximize his utility. However, it may be that the weighted income of all other individuals (b'/Y_j) are already so small that individual I does not desire to make a transfer to any of them. In fact, his utility would increase if any other individual transferred money to him. We assume, of course, that individual I cannot force any recipient to transfer money to him.

Under a voluntary transfer system, if the left hand side of equation 3.33 is initially greater than the right hand side, then individual I will transfer money to individual 1, the individual with the lowest income level, until 1's income is equal to individual 2's income. At that point, individual I will transfer money in equal amounts to both individuals 1 and 2. Individual I will proceed in this manner to raise the income floor until the right hand side of equation 3.33 is just equal to the left hand side.

The income floor nature of individual I's transfers is a direct result of the assumption that the coefficient b' (a preference weighting) is the same for all groups that enter individual I's utility function. (The coefficients for those income groups with incomes greater than Y_i are implicitly assumed to be zero by Von Furstenberg and Mueller.)

This brief analysis assumes that only individual I is making transfers. If some other individual makes transfers to one or more individuals whose incomes are less than individual I's, then individual I's decision calculus will change. He will be receiving benefits from another individual's transfers and he will not be charged for these benefits. In this case individual I will have an incentive to become a free rider. Both Hochman and Rodgers and Von Furstenberg and Mueller realize this. This is why they assume a collective organization with taxing powers as the vehicle for redistribution. Once they have set up this collective organization the relevant question becomes one of determining proper tax shares for various redistributive programs. Since we are currently concerned with the case in which there is only one potential donor, we shall ignore the possibility of a collective organization at this time.

If we allow donor preferences to be such that he does not weigh the incomes of all other individuals the same, then we can obtain transfer results that are not of the income floor variety. In the analysis that follows, we will show that the Von Furstenberg and Mueller model is a special case of a more general model and that with this more general model we can obtain transfer patterns consistent with those presented by Hochman and Rodgers. We propose to modify the Von Furstenberg and Mueller model as follows. The utility function for the donor will be given by:

$$U_1' = Y_1^{a} \prod_{i=2}^{N} \left(\frac{Y_i}{Y_1}\right)^{b_i} \tag{3.34}$$

or

$$U_1 = \log U_1' = (a - \sum_{i=2}^{N} b_i) \log Y_1 + \sum_{i=2}^{N} b_i \log Y_i. \tag{3.35}$$

By letting $b_1 = a - \sum_{i=2}^{N} b_i$ we can write

$$U_1 = \sum_{i=1}^{N} b_i \log Y_i. \tag{3.36}$$

We will let the utility functions of the N-1 non-donors be:

$$U_i' = U_i\ (Y_i) = Y_i^{a_i};\quad i = 2, \ldots, N. \tag{3.37}$$

Taking the logarithm of both sides yields

$$U_i = \log U_i' = a_i \log Y_i;\quad i = 2, \ldots, N. \tag{3.38}$$

It is assumed that:

$$Y_i = Y_i^* + S_i;\quad i = 1, \ldots, N \tag{3.39}$$

where Y_i^* is individual I's initial income and S_i is individual I's net transfer receipt. S_i will be nonnegative for all nondonors. It will be negative for individual 1, the donor, if he makes a transfer and zero if he does not.

If we assume second order conditions hold, then a utility maximum for the donor will occur when the total derivative of 3.36 is set equal to zero.

$$dU_1 = \sum_{i=1}^{N} \left(\frac{b_i}{Y_i}\right) dY_i = 0;\quad \sum_{i=1}^{N} dY_i = 0. \tag{3.40}$$

The donor's utility will be maximized when the gain in his utility due to the increased income of other individuals is just great enough to offset the loss in his utility due to his lowered income.

We choose to include the income levels of all individuals in our model rather than simply the income levels for those individuals whose incomes are less than the donor's because this is the more general formulation. If we continue to assume that individuals with lower incomes will never make transfers to those with higher incomes, then we need only assume that the marginal cost of making such a transfer is greater than the marginal benefit. We need not assume that the marginal benefit is zero as Von Furstenberg and Mueller assume. However, by including all individuals in the utility function, we leave open the possibility that we might observe transfers from poorer persons to richer persons.

The b_i are, in effect, preference weights that the donor has for individuals with income Y_i. In the collective aspects of the model, simplicity would dictate that the donor have the same b_i for all persons with incomes equal to Y_i. If we are looking at the purely voluntary aspects of the model this need not be the case. The donor could have differing b's for individuals with the same initial level of income due to the fact that 1) some of these individuals belong to his family; 2) some are his friends; 3) some belong to the same groups (race, sex, lodge, religion, business, etc.) that he belongs to or that he admires; 4) he is aware of their particular situation. As a matter of fact, the b's need not be equal in the collective aspect since a consensus among taxpayers may be that, given two groups of individuals with the same low income, transfers should be given to that group of individuals who are unable to work but not to the group whose members are capable of working but choose not to work. Taxpayer preferences and not income levels alone will determine who receives transfers and how great these transfers will be.

We will interpret the preference weights as follo s. If $b_i > 0$ then the donor will receive an increase in utility as recipient I's income increases. If $b_i < 0$ then the donor's utility will increase as individual I's income falls, and, so long as $b_i \leq 0$, the donor will never willingly transfer money to individual I. With $b_i < 0$, we can allow for envy or hatred on the part of the donor. If $b_i = 0$ then changes in individual I's income will have no effect on the donor's utility level; he will be totally indifferent toward individual I. The b_i are assumed constant during the period of analysis.

It might be assumed that the donor would weight smaller incomes more heavily in his utility function ($b_i > b_j$ for $Y_i < Y_j$), but if he does not wish to reverse the positions of any two income groups then b_i cannot be larger than b_j for $Y_i < Y_j$. If $b_i = b_j$ for $Y_i < Y_j$ then, given that transfers to individual I were sufficiently large, Y_i and Y_j would eventually merge into one larger income group. If the donor desires to strictly maintain the original income ordering, it is necessary that b_i be less that b_j when Y_i is less than Y_j. The fact that b_i is greater than zero does not indicate that the donor will necessarily transfer funds to individual I since the decision to transfer depends upon the ratios b_1/Y_1 and b_i/Y_i. If the latter is greater than the former a transfer will occur. If b_i/Y_i is greater than b_1/Y_1 and if Y_i is greater than Y_1, then the donor can increase his utility by making a transfer to individual I even though individual I has the larger income. Note that with this model it is not necessary that the lowest income individual receive a transfer even though some individual with a higher income does receive a transfer. If b_i/Y_i is greater than b_j/Y_j, b_j/Y_j is greater than b_1/Y_1 and Y_i is greater than Y_j, then the donor will transfer money to individual I before he transfers any to individual J even though J has the lower income.

Given a set of N individuals whose utility functions are given by equations 3.36 and 3.38 and whose initial levels of income are Y_1^*, Y_2^*, ..., Y_n^*, then with voluntary transfers the utility maximizing position for the donor (individual 1) will occur when

$$\sum_{i=2}^{N} \frac{b_i}{Y_i} dY_i = - \frac{b_1}{Y_1} dY_1 \tag{3.41}$$

and

$$d^2U_1 < 0. \tag{3.42}$$

As long as $\sum_{i=2}^{N} (b_i/Y_i)\, dY_i > -(b_1/Y_1)\, dY_1$, the donor will be able to increase his utility by transferring income to the individual with the largest b_i/Y_i. If b_i/Y_i is equal to b_j/Y_j and both are greater than b_K/Y_K and b_1/Y_1 for all $K \neq 1, i, j$, then the donor will transfer funds to both individuals I and J so as to keep b_i/Y_i equal to b_j/Y_j. The marginal transfer to individuals I and J will be divided so that the individual with the larger b will receive the larger share of the transfer. These shares will not be equal unless b_i is equal to b_j. The donor will continue making transfers until equation 3.41 holds. In order to get the income floor results obtained by Von Furstenberg and Mueller it is only necessary that 1) the donor have the same preference weights (b_i) for all individuals whose initial incomes fall below the desired income floor and that 2) the ratio b_i/Y_i for these low income individuals be larger than the ratio for higher income groups. In order to get the Hochman and Rodgers results of the donor making transfers to each of those below him on the income ladder such that the original income ordering is maintained, it is first necessary that the ratio b_i/Y_i be the same for all individuals whose incomes are lower than the donor's. It is not necessary that the ratios initially be the same, only that they are the same before he finishes making his transfers. Recall it is necessary that individuals with larger incomes are given a higher preference weighting if the income ordering is to be maintained. Second, it is necessary that the ratios be greater for those individuals whose incomes are less than the donor's than for those whose incomes are greater than the donor's.

This is a general model in that it allows us to explain various types of transfer behavior on the part of the donor. It can not only explain income floor type transfers and income differential type transfers, but it can also explain situations in which the donor makes transfers to individuals whose incomes are higher or the same as his own. It can explain his making transfers to individuals in one income class, while at the same time, he does not make transfers to individuals in a lower income class. It can also be used in explaining why he would want to make transfers to some individuals in a certain income class while not wanting to make transfers to other individuals in that same income class. We feel that the power this model has for explaining different types of transfer behavior makes it superior to other transfer models which have been proposed.

Looking back at equation 3.6, we may ask what changes we would have to make in our analysis if the good being transferred exhibited some public goods attributes. Suppose that two or more individuals could fully benefit from the transfer of the good in question as they might if it were a work of art. In this case the condition that is necessary for a transfer to be made is not

$$\frac{\partial U_1}{\partial X_{1j}} - [\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}] < 0 \quad \text{for some } i \tag{3.43}$$

but

$$\frac{\partial U_1}{\partial X_{1j}} - \sum_{i=2}^{P} [\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}] < 0, \tag{3.44}$$

where $P \leq N$ is the number of individuals who will benefit from the transfer.

The donor will take into account the benefits his transfer will give to all recipients who use the transfer. He will base his decision to transfer upon the cost to himself and the joint benefits. The cost to the donor may not be very large if the donor himself is a member of P, the group of individuals who utilize the transferred good. The fact that the donor transfers a good with public goods attributes is no reason why he too may not enjoy the benefits of consuming that good after the transfer has been made. This consumption before and after a transfer could occur, for instance, if an individual donated a work of art to a museum and then visited the museum after he had made the donation.

Also it should be noted that in equation 3.44 we have not multiplied the summed term on the left hand side by dX_{ij}/dX_{1j}. Since the good in question is assumed to have public goods attributes when transferred, we assume that

$$dX_{ij} = -dX_{1j} \quad \text{for all i contained in P.} \tag{3.45}$$

When the transferred good is public in nature, dX_{1j} is equal to $-dX_{ij}$ for all i in P, not $- \sum_{i=2}^{P} dX_{ij}$ as in equation 3.17 where the good was a strictly private good.

We have thus far assumed that both terms of $[\partial U_1/\partial X_{ij} + \partial U_1/\partial U_i \cdot \partial U_i/\partial X_{ij}]$ must be positive if a transfer is to be made. If they are negative for some individual I, then we assume that the donor does not like that individual. How will the existence of malevolent feelings affect the donor's transfer? When the good transferred is a strictly private good, the fact that the donor holds malevolent feelings toward one or more individuals will make no difference. He will make his transfers, if he makes any at all, to those individuals for whom he has benevolent feelings. He will not freely make any transfers to individuals he does not like.

If the good is a public good, however, the situation changes. If the donor is able to limit the number of individuals who are to receive his transfer, then he will take into account only the externalities caused by this limited number of individuals when he decides whether or how much to transfer. These limitations might take the form of specifications by the donor of who should benefit from the transfer. An example of this type of limitation is the Rhodes scholarship. Individuals who receive the scholarship must meet certain qualifications. For years one of the qualifications was that the recipient must be of the male sex. Whatever his motives, Cecil Rhodes was for years able to exclude women from sharing in his transfer.

The donor might also be able to take advantage of limitations which already exist within the society. For example, if a large number of the individuals the donor desires as recipients belong to a private club or organization, while a small number of individuals he does not desire as recipients belong to that same club or organization, then he can transfer the good to the club or organization. Once the transfer has been made, the good can be utilized by the many favored and a few disfavored members but not by the many disfavored nonmembers.

When the donor is not able to limit those individuals who will benefit from his transfer of a public good, he will sum all the externalities he faces, benevolent and malevolent. He will not make a transfer of the public good unless

$$\frac{\partial U_1}{\partial X_{1j}} - \sum_{i=2}^{N} \left(\frac{\partial U_1}{\partial X_{ij}} + \frac{\partial U_1}{\partial U_i}\frac{\partial U_i}{\partial X_{ij}}\right) < 0. \tag{3.46}$$

Note that even if the donor holds benevolent feelings toward most of the other members of his society, the existence of even one individual toward whom he holds strongly malevolent feelings can cause him to not make a transfer which he would have made in the absence of that individual. Likewise, the existence of one individual toward whom he holds strongly benevolent feelings can cause the donor to make a transfer of the public good even though he realizes that he does not like the majority of the individuals who will consume this public good.

Thus in deciding whether he should transfer a good with public qualities the transferor must take into account the externalities caused by all potential users of this public good. If the negative externalities outweigh the positive externalities, he will not make the transfer. If the positive externalities outweigh the negative ones then he may make the transfer if equation 3.46 holds. If he is able to consume some of the public good after he has transferred it, then his loss in utility from making the transfer will probably be small. This will increase the probability that he will make the transfer. If he is able to impose constraints on who can use the public good, then he will be able to eliminate all or some of the negative externalities caused by the consumption of the good by the "wrong" individuals. The ability to make limitations as to who can utilize the public good will also increase the likelihood that the transfer is made.

When there are more than two individuals in society exchange can take place between an individual who will receive or give a transfer and one who will not. What effect will such an exchange have on the number of transfers and on the level of transfers which take place? The answer is dependent upon the timing of exchanges vis-a-vis transfers. The number and level of transfers will generally be greater if they occur before rather than after exchange has taken place.

Insofar as the interdependency in the donor's utility function is caused wholly or partially by "utility" externalities, exchange will cause the level of transfers to fall. An individual will only enter into an exchange if the exchange leaves him no worse off. Generally the individual will expect to be better off. If exchange increases his utility level, the marginal benefits available to the potential donor from making the transfer will fall. This will decrease the optimal size of the donor's transfer and may even eliminate the transfer all together.

If the interdependency is caused wholly or partially by a "goods" externality, then the direction of trade for that good will determine whether the incentive to transfer increases or decreases. If the "goods" externality is the result of the recipient's low possession of that good, then he may try to get more of that good through exchange. That is, the recipient's own marginal utility for the good might be relatively high. If he does increase his holdings of the good through exchange, there will be less incentive for the donor to transfer that good. If the potential recipient trades away the relevant good during exchange, then the chances for a transfer of that good will be increased.

If both types of externality are present and the recipient exchanges other goods for the relevant good, then the donor's marginal benefit from making the transfer will fall. If he trades away the relevant good the change in the donor's marginal benefit is indeterminate since $\partial U_1/\partial U_2$ will fall due to the recipient's (individual 2's) increased utility level and both $\partial U_1/\partial X_{21}$ and $\partial U_2/\partial X_{21}$ will rise due to the recipient's lower holdings of that good.

Suppose the donor also engages in pre-transfer exchange. His decision to transfer is based upon marginal benefit $(\partial U_1/\partial X_{21} + \partial U_1/\partial U_2 \cdot$

$\partial U_2/\partial X_{21}$) minus marginal cost ($\partial U_1/\partial X_{11}$). If his marginal cost for a good is low, he will probably trade away some of that good during exchange. This will raise his marginal utility for that good and raise the marginal cost of making the transfer. Thus, if he trades the relevant good away, he will reduce his incentive to transfer. If he trades for the relevant good, then the marginal cost of his transfer will fall, increasing the likelihood of a transfer. Whether the donor's desire to make a transfer increases or decreases after exchange depends upon what happens to both his marginal cost and marginal benefits. Generally we would expect transfers to be reduced or eliminated after exchange but this will not always occur.

If transfers take place before exchange, then the transfer decision will be based upon marginal conditions in force after the original distribution. We know from Chapter II that transfers will not eliminate the desire or need for exchange. A Pareto efficient point will not be reached through transfers. A question arises. If transfers precede exchanges, will the resulting exchanges lead to situations where further transfers are desired? In the two person case the answer is no. The initial transfer leads to a situation where the donor's marginal rate of commodity substitution (MRCS) is zero, infinity, or undefined. His MRCS is undefined when the donor has reached his bliss point. If the donor has not reached his bliss point, exchange will occur until the contract curve is reached. In the two person case, post-transfer exchange will lead to the Pareto efficient locus.

In the many person case this result need not hold. Suppose there are three individuals in the society, the donor (individual 1), the recipient (individual 2), and a third individual who does not enter the donor's utility function. In the post-transfer situation the donor's MRCS will be zero or infinity with respect to the recipient. If the donor and recipient trade, the donor must exchange the transferred good (good 1) for another good. This will lessen his incentive to make further transfers of good 1.

If the recipient makes exchanges with the third individual, then the donor's incentive for making further transfers will depend upon the direction of exchange for good 1. When the recipient trades for good 1, the donor will lose incentive for making further transfers since both $\partial U_1/\partial X_{21}$ and $\partial U_1/\partial U_2 \cdot \partial U_2/\partial X_{21}$ will fall. If the recipient trades away good 1, then the donor may have increased incentive to make further transfers. His incentive will increase if $\partial U_1/\partial X_{21}$ rises more rapidly than $\partial U_1/\partial U_2 \cdot \partial U_2/\partial X_{21}$ falls.

If the donor makes exchanges with the third individual his MRCS is no longer zero or infinity. Recall that the donor's MRCS for two goods is:

$$\frac{\frac{\partial U_1}{\partial X_{11}} + [\frac{\partial U_1}{\partial X_{21}} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_{21}}]\frac{dX_{21}}{dX_{11}}}{\frac{\partial U_1}{\partial X_{12}} + [\frac{\partial U_1}{\partial X_{22}} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_{22}}]\frac{dX_{22}}{dX_{12}}} . \tag{3.47}$$

If the donor and the nonrecipient engage in exchange, $dX_{2i} = 0$ $(i = 1, 2)$ so that the terms in brackets fall out. This means that when the donor trades with the nonrecipient he is acting as an economically selfish individual. If he trades for the transferred good $\partial U_1/\partial X_{11}$ will fall, lowering the costs of making further transfers of that good to the recipient. If he trades away good 1, $\partial U_1/\partial X_{11}$ will rise, increasing transfer costs. The donor will not have an incentive for making further transfers of good 1 if he trades away that good during the exchange process and the recipient trades for that good. Any other exchange pattern (both trade for the good, both trade it away, or the donor trades for the good and the recipient trades it away) may lead to incentives for further transfer of good 1. It should be noted that post-transfer exchange may lead to situations in which the transfer of goods not previously transferred becomes desirable.

In this section we have looked at a situation in which there is one donor and N-1 potential recipients. We have found that the donor's behavior will depend upon the nature of the good being transferred. If the good is a strictly private good, then the donor will act as in the two person case. If there is one individual toward whom the donor is so benevolent that a transfer of some units of the good to that individual increases the donor's utility, the donor will transfer the good to that individual. The donor will continue transferring the good in question until the loss in his utility caused by the reduction in his consumption of that good is just equal to the increase in his utility caused by the recipient's increased consumption of that good. His transfer will go to only one individual unless the increase in his utility caused by this recipient's consumption of the last unit of the good was equal to or less than the increase he could obtain by transferring the next unit of the good to another recipient. He will continue to transfer to the two or more recipients until the marginal cost of his transfer equals the marginal benefits.

If the good to be transferred is a public good, then the transfer decision will depend in part upon the donor's ability to exclude undesirable individuals from consuming the good. In any case, the donor will sum the "goods" and "utility" externalities caused by all the potential recipients of the public good and will compare this sum with the cost of making the transfer. If all externalities are benevolent, the donor will have no reason to limit those who can receive his transfer. If some are malevolent externalities, however, limiting the transfer to those individuals who exert only benevolent externalities on the donor will increase the chances of the transfer being made.

We also found that when interdependent utility functions are present the system will not necessarily reach an equilibrium after one round of transfer-exchange or exchange-transfer. Transfers necessarily lead to a situation where further exchange is desirable. Exchange, while generally dampening the incentive for transfers, will occasionally increase the desire for further transfers.

Several Donors

In this section we make the same basic assumptions as in the last section. The notable difference is that there are now $K > 1$ potential donors instead of one. All potential donors are assumed to have the same type of interdependent utility function as that given in equation 3.1. The utility function for all nonpotential donors is given by equation 3.2. Our assumptions concerning fixed quantities of goods per period of time, rationality of consumers, nonappropriation, and costless exchanges and transfers remain in effect. We will assume that all relevant second order conditions are always satisfied. We will also henceforth assume that when a potential donor is in a region of possible charity with respect to a certain good and a certain potential recipient he will freely make a transfer of the appropriate amount of the good in question to the recipient individual. This assumption is made so that we do not have to continually remind the reader that in a region of possible charity the donor will either make a free transfer of the good or offer that good to the recipient at terms of trade better than those available on the market.

We will look at seven different models when analyzing the case of several donors. We will move from the simplest case in which the recipients of each donor are not potential recipients of other donors to the most complex case in which every individual in the society, donors and recipients alike, has an interdependent utility function.

Case I: Mutually exclusive recipients

In this first case it is assumed that there are $K > 1$ donors and N-K potential recipients. These N-K potential recipients are divided into K subsets such that each donor has a separate and distinct subset of recipients. Thus there are K groups or tribes, each with its own benefactor or patron. Each benefactor will independently decide whether he should make a transfer to his recipients. He will follow the procedure outlined in the first section of this chapter in determining who should receive transfers. Each benefactor will make transfers to all of his potential recipients, to a subset of them, or to none of them depending upon the strengths of the externalities affecting him and the type of good being transferred. Since we have already discussed the major points of this case in the last section we will proceed to case II.

Case II: Common recipients

In this second case we again assume that there are K potential donors and N-K potential recipients. While the welfare and consumption levels of some potential recipients may be of concern to only one donor, this is not true of every recipient in the society. We assume that there are some potential recipients whose consumption and welfare levels impose externalities on two or more potential donors. In this subsection we will study the effects that common recipients will have on the transfer behavior of two or more donor individuals.

For simplicity we will work with a model in which there are only three individuals. Two have interdependent utility functions and are potential donors. The third has a strictly private utility function but enters the utility function of each potential donor. Neither potential donor enters the utility function of the other potential donor. The utility functions of the three individuals will look as follows:

$$U_1 = U_1(X_1, X_3; U_3), \tag{3.48}$$

$$U_2 = U_2(X_2, X_3; U_3), \tag{3.49}$$

and

$$U_3 = U_3(X_3), \tag{3.50}$$

where X_i is the consumption bundle for the i^{th} individual ($i = 1, 2, 3$).

The first order conditions for utility maximization are:

$$dU_1 = 0 = \frac{\partial U_1}{\partial X_1} dX_1 + \frac{\partial U_1}{\partial X_3} dX_3 + \frac{\partial U_1}{\partial U_3} \frac{\partial U_3}{\partial X_3} dX_3, \tag{3.51}$$

$$dU_2 = 0 = \frac{\partial U_2}{\partial X_2} dX_2 + \frac{\partial U_2}{\partial X_3} dX_3 + \frac{\partial U_2}{\partial U_3} \frac{\partial U_3}{\partial X_3} dX_3, \tag{3.52}$$

and

$$dU_3 = 0 = \frac{\partial U_3}{\partial X_3} dX_3. \tag{3.53}$$

We have already assumed that the second order conditions for utility maximization are met.

If both donors are malevolent toward individual 3 no problems arise. Neither potential donor will make a transfer to individual 3 and, since we have assumed nonappropriation, neither will be able to increase his utility by taking goods from individual 3.

When one potential donor, say individual 1, holds malevolent feelings toward the potential recipient and the other has benevolent feelings strong enough to cause a transfer, the transfer will be made. Individual 1 may try to persuade individual 2 to reduce his transfers to the recipient, but individual 2 will ignore this request since the level of individual 1's utility is of no concern to him. Individual 1 cannot bribe individual 2 to reduce his transfer since any such bribe will increase individual 2's bundle of goods. This will lower individual 2's marginal utility from consumption and increase his desire to make transfers to individual 3. Individual 1 cannot threaten individual 2 since we have assumed nonappropriation which rules out lowering another individual's utility level through forceful actions. Thus if individual 2 desires to make a transfer to individual 3 he will make the transfer without concern for the negative effects this transfer causes individual 1. Individual 1 will have to live with the transfer and with a lowered level of utility.

The above situation presents some interesting insights into the actions and behavior of individuals. When all utility functions are strictly private, the exchange of goods between two individuals will not affect the utility levels of any other individual. Since no one is made worse off by the exchange, the post-exchange distribution of goods must be Pareto better than the pre-exchange distribution. When utility functions are interdependent and some of the externalities are malevolent, all transfers and/or exchanges may not lead to Pareto better situations. The two individuals involved in the transfer or exchange will be better off, but other individuals may be made worse off.

The individuals who would be made worse off by a transfer have an incentive to try to stop the transfer or exchange or to at least try to reduce the amount transferred or exchanged. Thus, individuals who meddle in situations which do not seem to concern them may, in fact, be very concerned. Their involvement may be very rational from an economic standpoint.

If enough individuals experience negative externalities from the consumption or welfare levels of other individuals then it may be possible that no move from the initial distribution of goods will be a Pareto better move. It may be possible that <u>any</u> transfer or <u>any</u> exchange will make at least one individual worse off. In this case the Pareto criterion is no longer useful for determining society's preferred distributions of goods. Some method capable of making interpersonal utility comparisons is needed.

An interesting situation arises when both individuals 1 and 2 have benevolent feelings for individual 3. Suppose that the external effects are so strong that both potential donors desire to make a transfer to individual 3. If each acted without regard to the actions of the other, the resulting total transfer to individual 3 might be larger than either of the donors desired. It certainly would be larger than the transfer each desired to make (and made) when he acted alone. This larger than desired transfer is caused by the fact that each donor acts as if he is the only transferor whereas he is really one of several transferors. That is, each acts as if $dX_3 = - dX_i$ $(i = 1, 2)$ whereas dX_3 is really equal to $- (dX_1 + dX_2)$.

We have assumed that the potential donors have some degree of accurate knowledge of the recipient's holdings of goods and of how changes in these holdings affect the recipient's utility levels. With such knowledge it would not be long before each donor realized that the recipient was receiving transfers from other donor individuals. The knowledge that a recipient is receiving transfers from more than one source may influence some of the donors to become free riders. That is, they will stop their own transfers to the recipient with the knowledge that other donors will continue to make transfers. The free riders will receive increases in their utility levels since the recipient will be better off, but this increased utility will have cost them nothing since they made no transfers themselves.

Suppose individual 1 in our model attempts to free ride. Given that individual 2 continues his transfers to individual 3, the utility levels of all three individuals will increase. Individual 3's utility will increase since he has a larger bundle of goods. Individual 2's utility will increase since he has made transfers until

$$\frac{\partial U_2}{\partial X_2} - [\frac{\partial U_2}{\partial X_3} + \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial X_3}] = 0; \quad dX_3 = -dX_2. \tag{3.54}$$

Individual 1's utility will increase even though he made no transfer since he is benevolent toward individual 3 and both 3's holdings of goods and his utility have increased due to the transfer from individual 2. Note that if individual 2's transfers are not sufficient to cause $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$ to fall below $\partial U_1/\partial X_1$, then individual 1 will make additional transfers to individual 3 until these two terms are equal.

Figure 3.1 is a graphical representation of individual 1's situation. The horizontal axis measures the amount transferred by individual 1 and/or to individual 3. The vertical axis measures individual 1's gains or losses in marginal utility. The MC curve is individual 1's marginal cost of making a transfer and is equal to $\partial U_1/\partial X_1$ in equation 3.51. The MB curve measures the marginal benefits received by individual 1 when individual 3 receives a transfer. This is the $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$ term in equation 3.51. If individual 1 makes the only transfer to individual 3, he will transfer until his marginal benefits equal

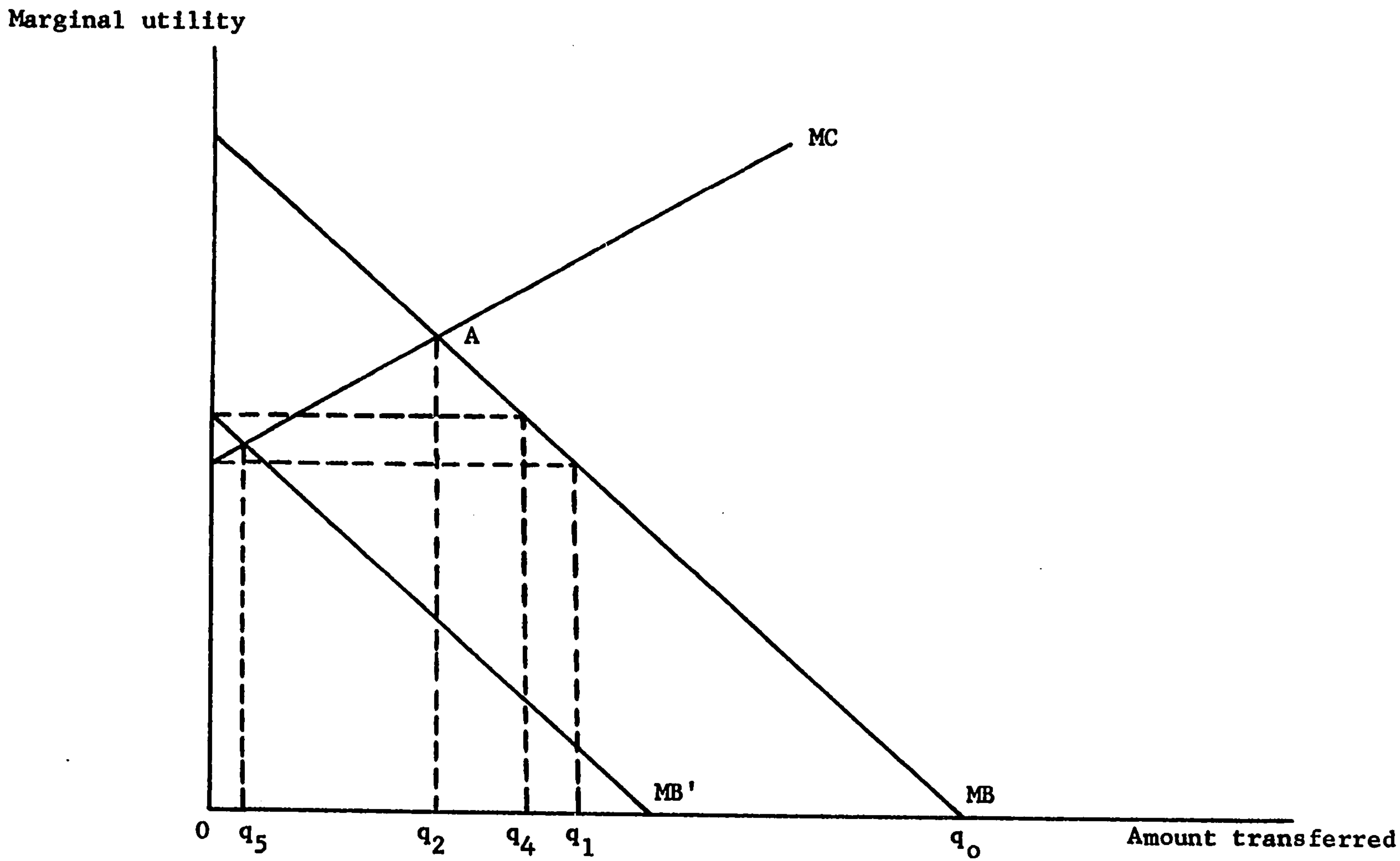

Figure 3.1. A donor's marginal cost and marginal benefit from a transfer

his marginal costs (point A). Individual 1 will transfer q_2 units of the goods bundle at this optimal transfer point.

If individual 1 does not make a transfer but individual 2 does, then the increase in individual 1's utility will be equal to the relevant area under the MB curve. Individual 1's utility will be maximized if individual 2 transfers q_o units of the bundle to individual 3 for at this point individual 1's marginal benefit from further transfer is equal to zero. Individual 2 will only transfer until _his_ marginal benefits from the transfer are equal to _his_ marginal costs. Assume that individual 2's optimal transfer is less than q_o. If his optimal transfer is greater than or equal to q_1 individual 1 will not have the incentive to make further transfers to individual 3 since his marginal benefits from making the transfer will be no greater than his marginal costs. If individual 2's optimal transfer level is less than q_1, say q_4, then individual 1 can increase his utility by making a further transfer, of q_5, to individual 3. The MB' curve in Figure 3.1 is the marginal benefit curve facing individual 1 after individual 2 has transferred q_4 units of the goods bundle to individual 3.

It is easy to see from Figure 3.1 that individual 1's utility will increase most rapidly if he does not contribute to the transfer received by individual 3. He will receive the benefits of the transfer but will incur none of its costs. As long as individual 2 transfers at least q_1 units of the goods bundle to individual 3, individual 1 will have no incentive to make further transfers to individual 3. He will become a free rider.

One way to overcome the free rider problem is for the two donors to jointly administer the transfer. There are two problems which must be solved when a joint transfer is made. The first problem is deciding on the total size of the transfer. The second is deciding on the appropriate shares to be borne by each donor. The Lindahl solution to the public goods problem provides a solution to these problems of joint transfer [33]. With the Lindahl solution, both donors will end up in a post-transfer situation where

$$\frac{\partial U_i}{\partial X_i} dX_i + [\frac{\partial U_i}{\partial X_3} + \frac{\partial U_i}{\partial U_3} \frac{\partial U_3}{\partial X_3}] dX_3 = 0; \qquad (3.55)$$

$i = 1, 2$ and $dX_3 = -dX_1 - dX_2$.

In the Lindahl solution it is assumed that there is a continuous function which relates an individual's desired size of the total transfer with his share of that total transfer. If his share of the transfer is zero, then the maximum transfer he desires is the one which sets $[\partial U_i/\partial X_3 + \partial U_i/\partial U_3 \cdot \partial U_3/\partial X_3]$ equal to zero where i represents this particular individual. This amount corresponds to q_o in Figure 3.1. When he makes the entire transfer, the maximum transfer he will make is the one which sets $-[\partial U_i/\partial X_3 + \partial U_i/\partial U_3 \cdot \partial U_3/\partial X_3]$ equal to $\partial U_i/\partial X_i$ (i.e., q_2 in Figure 3.1). This desired transfer will necessarily be smaller than the preceding one where he made no contribution to the transfer. If the individual's share of the transfer is between zero and one, then the desired maximum transfer will be between the two extremes mentioned above. As an individual's transfer share is increased, his desired maximum transfer decreases.

In Figure 3.2 we have drawn individual 1's Lindahl curve. The vertical axis in Figure 3.2 measures the desired level of transfer while the horizontal axis measures the individual's share of the transfer to individual 3. This share increases from zero on the left boundary to one on the right. The size of individual 1's desired transfer when he does not contribute to the transfer is q_o. When he makes the entire transfer the size desired is q_2.

Individual 1's utility will increase as he moves along the curve from q_2 to q_o. There are two reasons for this increase in his utility. First, the recipient will receive more goods so individual 1 will receive a larger benefit from the transfer. Second, the donor will bear a smaller share of the transfer. The size of his transfer may initially increase as his share of the total transfer declines. This will cause the marginal cost of the transfer to increase. The slope of his Lindahl curve will determine whether the size of his transfer will initially increase or decrease as his share of the transfer decreases. The steeper the slope of the Lindahl curve the more likely the size of his transfer will increase before it decreases. In Figure 3.2 the $q_2 0$ curve shows the size of the transfer individual 1 is willing to make given his share of the transfer and the size of his desired transfer. Even though the marginal cost of his transfer may be increasing his utility will still increase since his marginal benefits from the total transfer will increase even faster.

If another donor makes a transfer to individual 3, individual 1's Lindahl curve will fall. If individual 2 made a transfer of q_1 as in

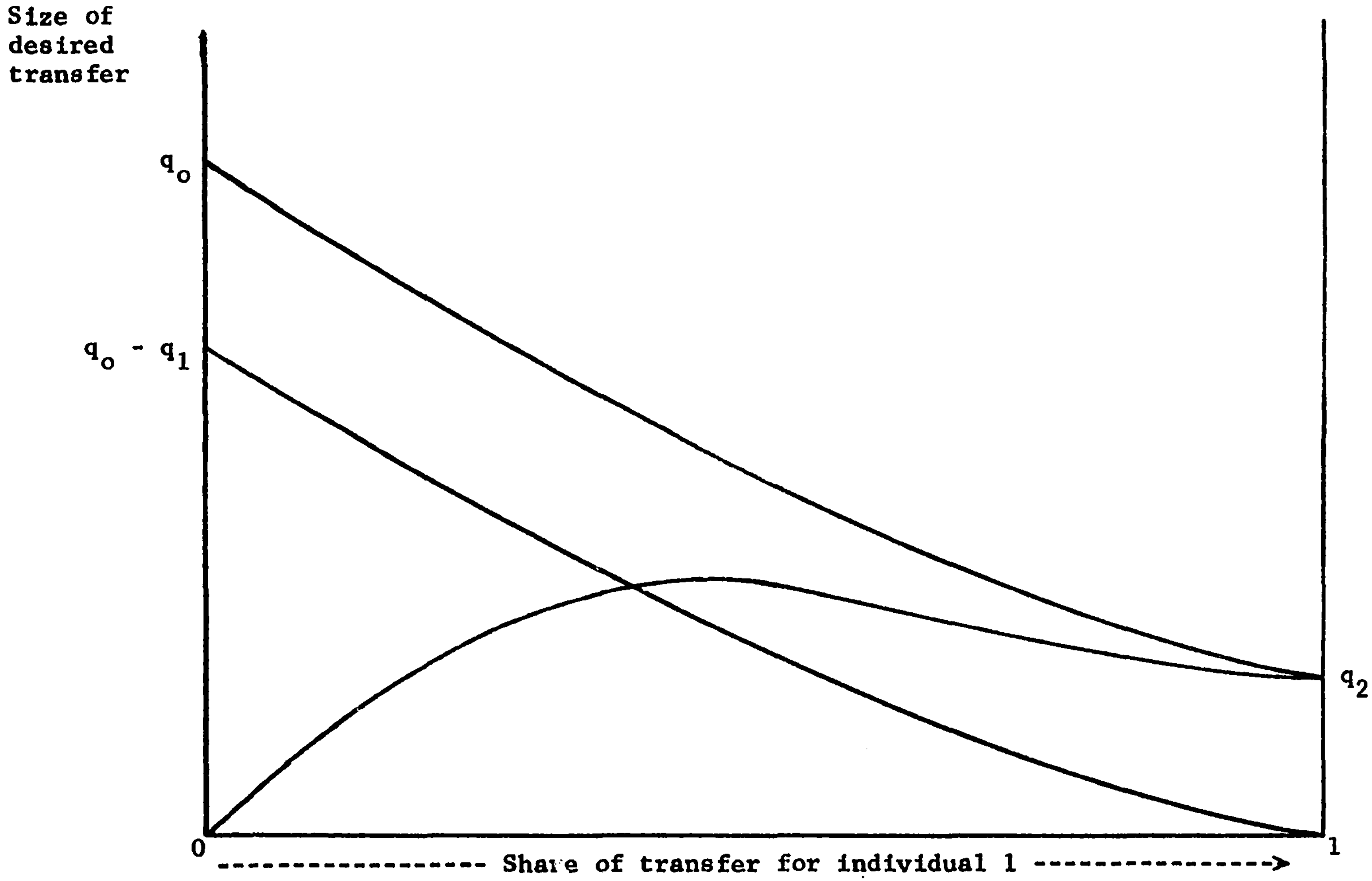

Figure 3.2. The Lindahl curve

Figure 3.1, individual 1 would just lose the incentive to make a further transfer to individual 3. His Lindahl curve would shift down from q_2q_o to $1,(q_o-q_1)$. He would still be willing to make a joint transfer but he will no longer make a transfer alone.

The transfer by individual 2 will increase individual 1's utility. Figure 3.3 shows three different utility curves for individual 1. Utility is measured on the vertical axis and the size of the transfer to individual 3 is measured on the horizontal axis. The solid curve from $U_{initial}$ through the point A (U_A) is individual 1's utility curve when he is the only one making transfers. His utility is maximized at point A (U_1) after he transfers q_2 units of the goods bundle to individual 3.

The utility curve from $U_{initial}$ through the point B (U_B) is individual 1's utility curve when he makes no contribution to the transfer. U_B is always above U_A and reaches a maximum at point B when q_o units of the goods bundle have been transferred to individual 3. At point B individual 1's utility level is $U_{maximum}$. This is the maximum level of utility he can achieve through transfers alone. We have drawn U_B so that individual 1's utility is constant at $U_{maximum}$ if more than q_o units are transferred, but it would decline after point B if a transfer greater than q_o caused individual 1 to become malevolent toward individual 3. Too much of a good thing could be bad for individual 1.

The third curve in Figure 3.3 (U_C) represents individual 1's utility level as he moves along his Lindahl curve. U_C lies below U_B since individual 1 is incurring some marginal cost when he shares in a transfer of a given size. The greater the marginal cost of making a transfer,

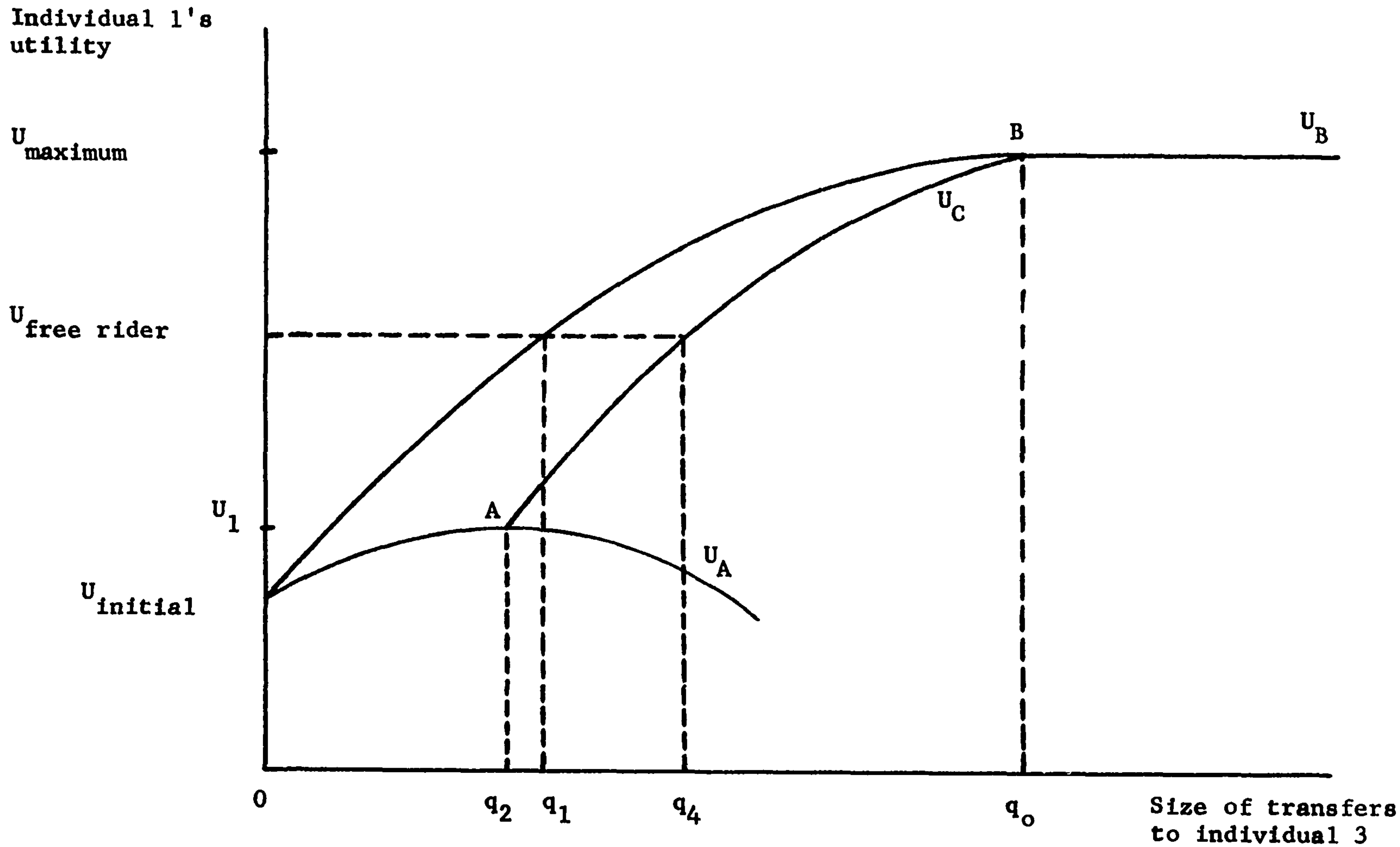

Figure 3.3. The donor's utility curves when he makes a transfer alone (U_A), when he does not share in the cost of the transfer (U_B), and when he shares in a joint transfer (U_C).

given the marginal benefits, the farther U_C will generally be from U_A.

In Figure 3.4 we have drawn the Lindahl curve for individuals 1 and 2. Individual 1's share of the transfer increases and individual 2's decreases as we move from left to right. If individual 1 makes the entire transfer he will make a transfer of size q_2 whereas individual 2 desires a transfer of q_5. Likewise if individual 2 makes the only transfer he will make a transfer of q_1 while individual 1 desires a transfer of q_0. Point D in Figure 3.4 is the only point where the two individuals will agree both on the size of the transfer to be made (q_3) and upon the share to be borne by each (α for individual 1 and $(1-\alpha)$ for individual 2).

If a joint transfer is made it will be of size q_3. If individual 1 decides to become a free rider the transfer size will be q_1. This is the amount individual 2 is willing to transfer to individual 3 when he makes the transfer alone. The question at hand is will individual 1 make a joint transfer with individual 2 or will he become a free rider. To find the answer we must look back at Figure 3.3. When individual 1 is a free rider, q_1 will be transferred and his utility will be $U_{\text{free rider}}$. For individual 1 to be willing to make a joint transfer he would have to have a utility level higher than $U_{\text{free rider}}$. This will happen, given his particular Lindahl curve, when the joint transfer is greater than q_4. Thus q_3 in Figure 3.4 must be greater than q_4 in Figure 3.3 if individual 1 is to willingly share in the transfer to individual 3. We assume, of course, that the q_i's ($i = 0, 1, 2, 3, 4$) in Figures 3.3 and 3.4 represent the same quantities of the goods bundle.

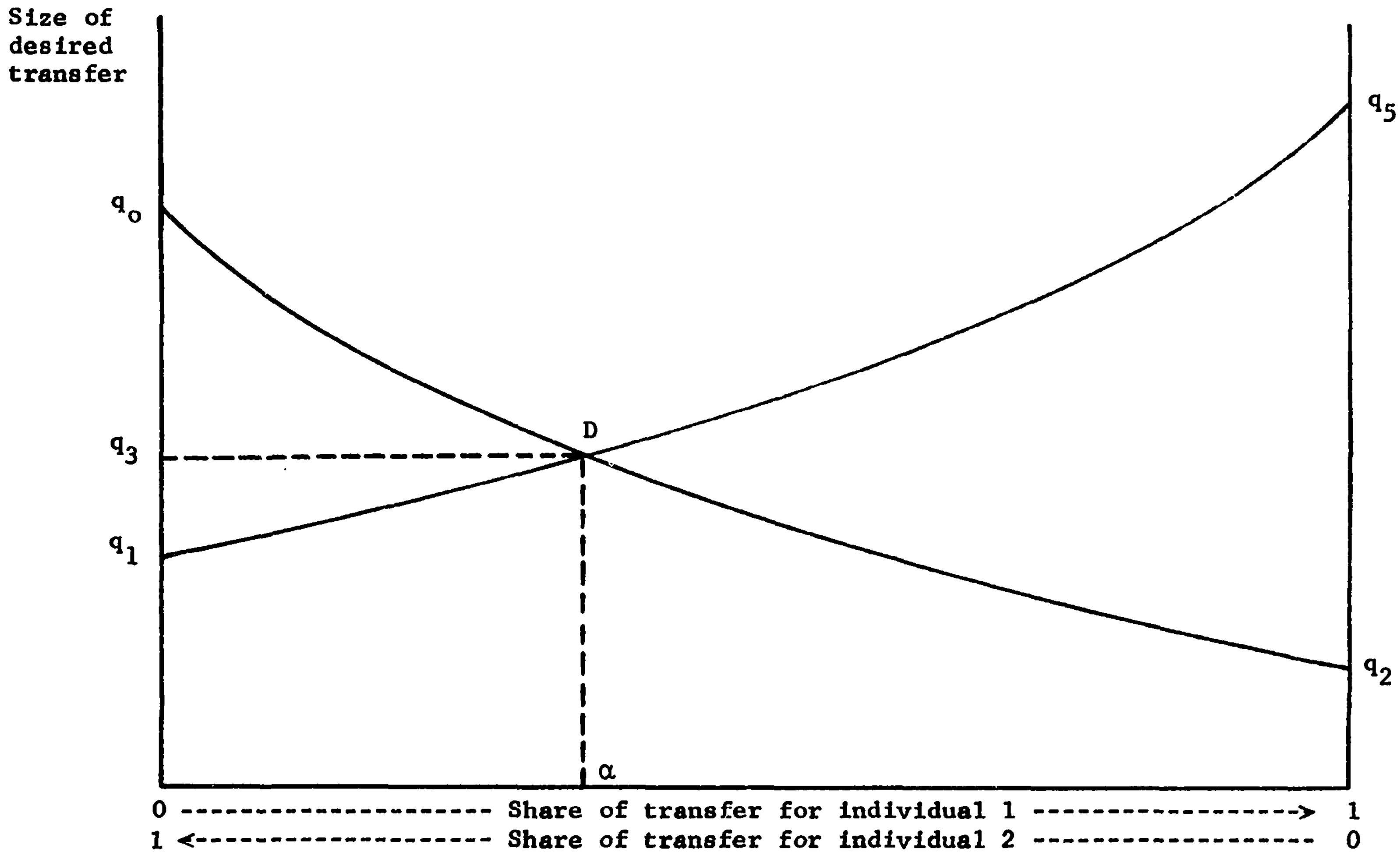

Figure 3.4. The Lindahl solution

Suppose that both potential donors were benevolent toward the potential recipient, but that only one of them, say individual 2, faced an externality great enough to make him transfer goods to the recipient. It may seem that individual 1 would never have an incentive to make a transfer since the transfer from individual 2 to individual 3 will further reduce the marginal benefit he would get from making a transfer. Thus it may seem that individual 1 will always be a free rider. However, it may be possible for individual 2 to convince individual 1 to make a joint transfer. Individual 1 might agree to the joint transfer if, when his transfer share is zero, he desires a greater transfer for the recipient than is made by individual 2. This can be seen in figure 3.5.

Figure 3.5 shows that individual 1 is not willing to make a transfer to individual 3 unless his share of that transfer is less than β. He will never voluntarily make a transfer if he has to finance it entirely himself. In this instance, $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$ is positive but less than $\partial U_1/\partial X_1$. Note that q_o is greater than q_1. The transfer by individual 2 does not set $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$ equal to zero. Thus there must be some size transfer between q_o and q_1 where the two donors will agree on relative shares. This size transfer is, of course, q_3. The share for individual 1 is α and the share for individual 2 is $1 - \alpha$.

Again, the fact that the two potential donors agree on the size and shares of a joint transfer does not imply that such a joint transfer will take place. The joint transfer will occur only if individual 1 receives a greater increase in his utility from making a joint transfer than from

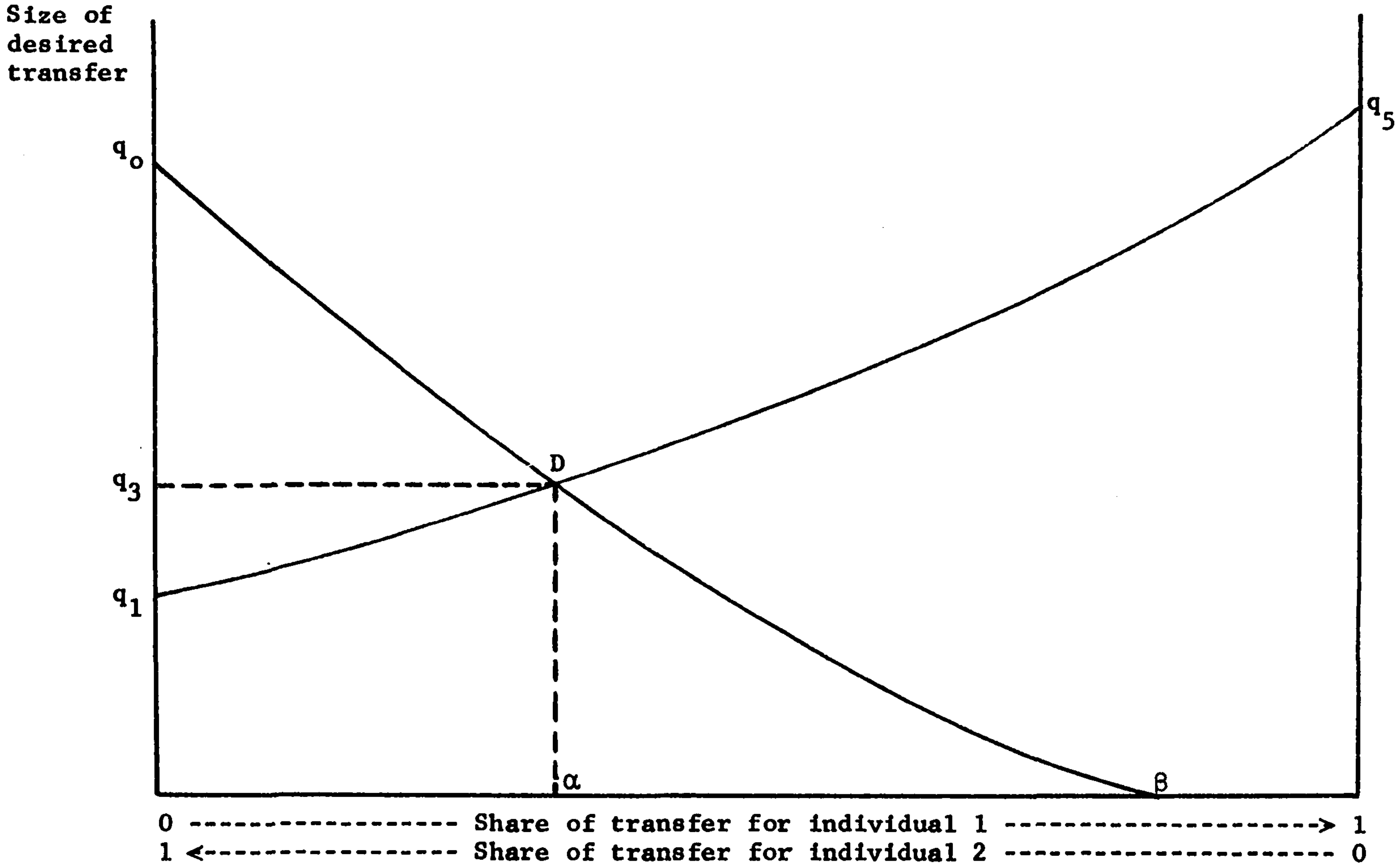

Figure 3.5. The Lindahl solution when one donor refuses to be the sole donor

being a free rider. This, as we have seen before, will depend upon the values of q_1, q_3, and the size of $\partial U_1/\partial X_1$ given $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$. In general, if individual 1 is not willing to make a transfer on his own, we assume that he will only agree to join in the joint transfer if his share of such a transfer is very small.

In Figure 3.6 we see a situation in which no joint transfer will be made even though individual 1 has benevolent feelings for individual 3. The reason no joint transfer will be made is that the externality on individual 1, caused by individual 3, will disappear by the time individual 2 finishes making his transfers. That is, $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$ will become zero. Individual 1 might be willing to bear a share of the transfer if his share is less than β but the two individuals will never be able to agree on a desired transfer size.

Suppose the two donors are benevolent toward the recipient but neither desires to make a transfer on his own. They will be willing to make a joint transfer if their Lindahl curves are similar to those depicted in Figure 3.7. The necessary condition for a joint transfer to be made in this case is that the Lindahl curves for the two individuals cross at a positive desired transfer size. Figure 3.7 shows a transfer of q_3. Since neither potential donor would make a transfer when acting along neither can be a free rider. Since they can both increase their utility by making a joint transfer and since we have assumed that they are both rational consumers, both will join in making the joint transfer.

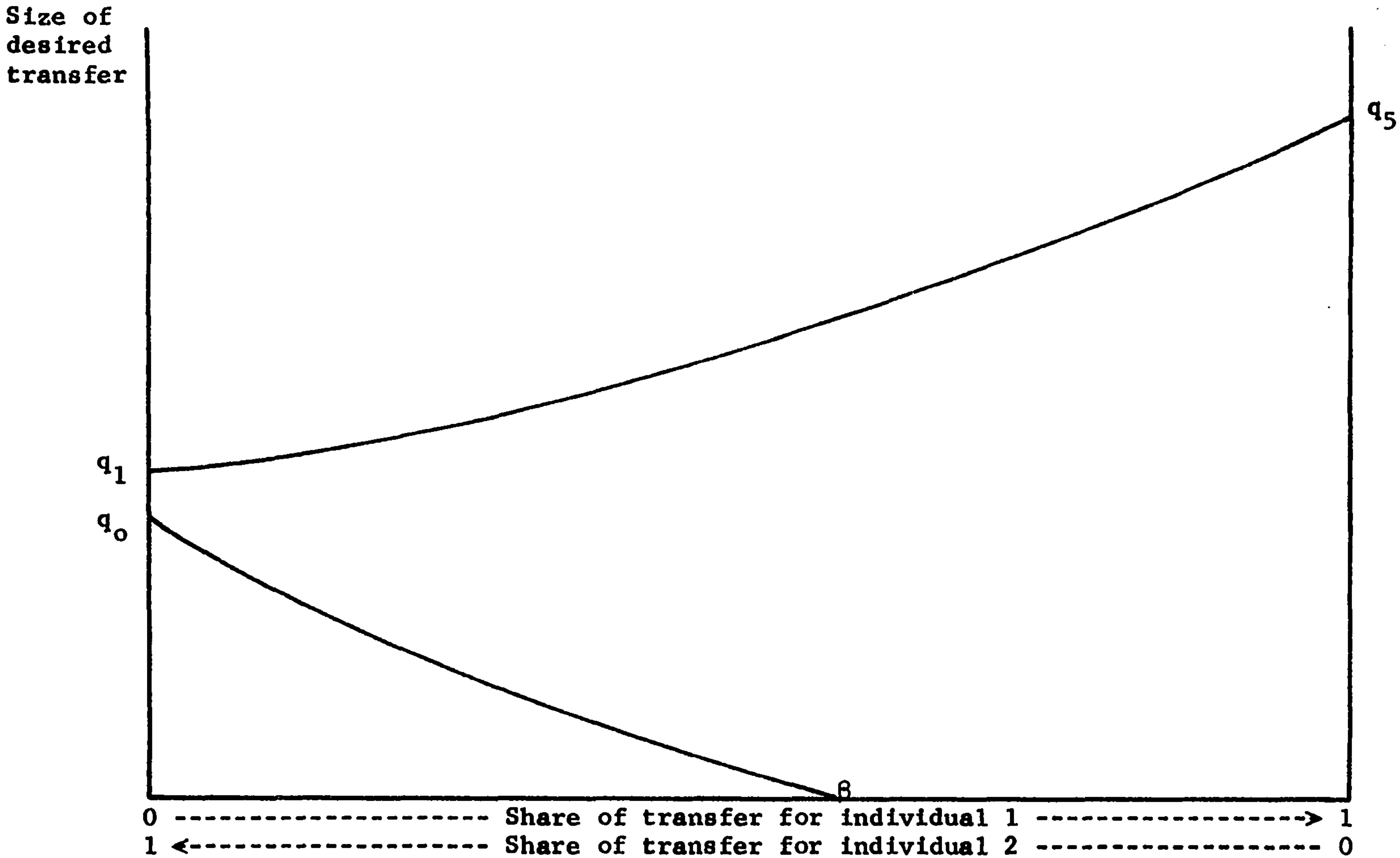

Figure 3.6. A situation where no joint transfer will occur

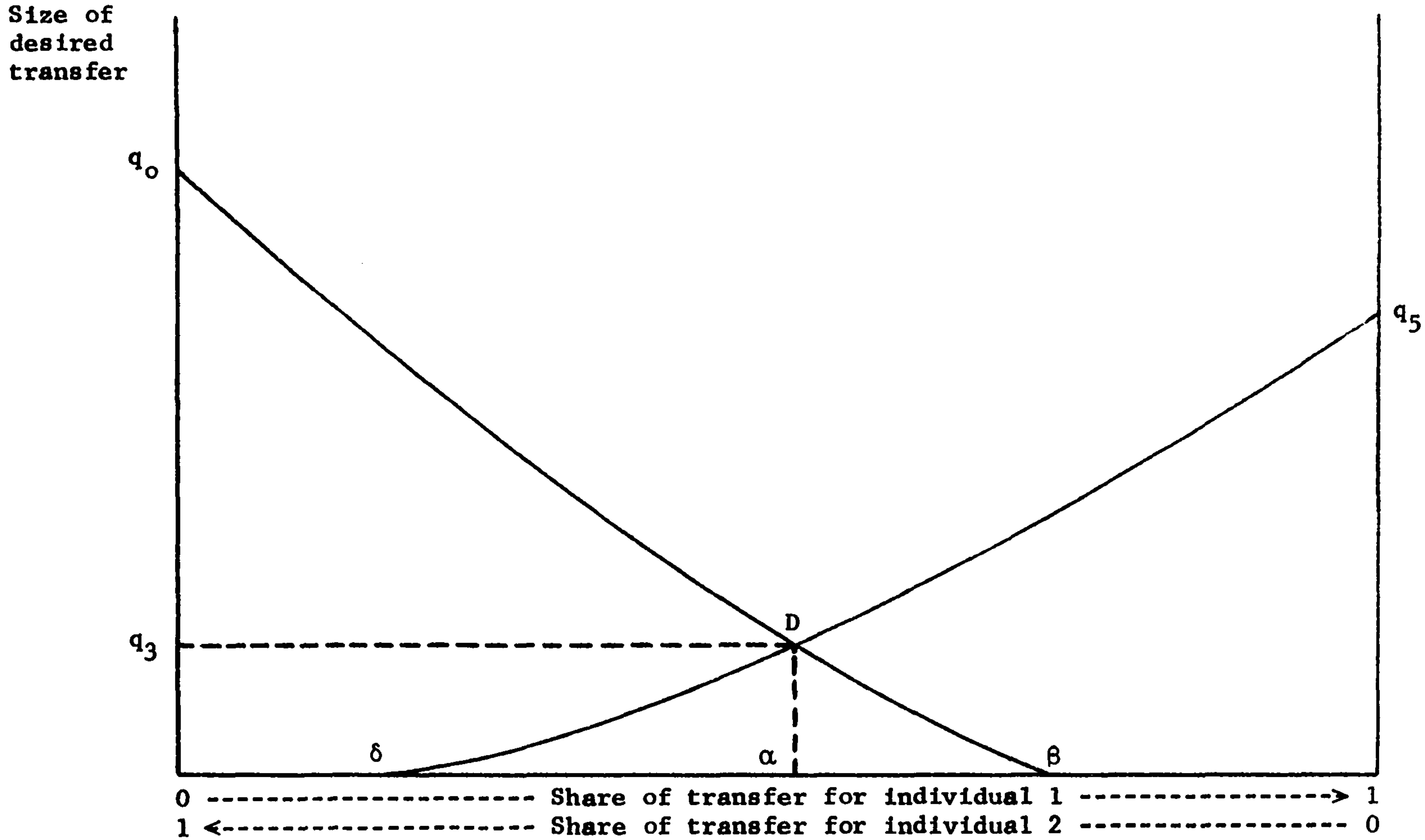

Figure 3.7. The situation where a joint transfer will be made even though neither donor would make the transfer alone

This type of consumer behavior may be one rationale for the formation of charitable organizations. Individuals who would not make a transfer to a person or organization if they were the only ones making the transfers might make a contribution if they knew that this contribution was just a small portion of the total transfer.

The free rider problem will probably be greater when there are many potential donors than when there are only a few.[1] In the first place there will be some individuals whose situation is similar to that depicted for individual 1 in Figure 3.6. These individuals might be willing to share in a transfer but not in the amount proposed by others. In the second place, the greater number of potential donors means there will be a greater number whose utility increases if they participate in a joint transfer but whose utility does not increase as much as it does when they are free riders. On the other hand, the greater number of potential donors would indicate that each would have a small share of a large transfer. The smaller an individual's share and the larger the total transfer the greater will be the incentive to join in the joint transfer. The final decision on whether or not a potential donor will assist in the joint transfer will depend on how much his utility increases when he joins the group versus how much it increases when he is a free rider.

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[1]For an excellent treatise on this point see Mancur Olsen [40].

It has been argued that in small sized groups social pressure will help reduce the free rider problem [24; 26; 28]. As the size of the group increases, social pressure becomes less powerful in dealing with free riders. This is a valid argument and there is some evidence which supports it [34]. Social pressure increases the costs of certain actions or certain nonactions. In this case it increases the costs of being a free rider. Social pressure based on threats or penalties is a form of appropriation and we have assumed nonappropriation. Therefore, we shall assume away this type of social pressure for the remainder of the paper.

The potential free rider problem has been cited by some authors as a reason why transfers should be funneled through a governmental agency [8; 22; 38; 49]. The difference between this type of collective action and the voluntary collective action described earlier is that the government has the power of taxation. (Forget for the moment that taxation is probably the ultimate form of appropriation.) With public collectivization, the government can supposedly determine each individual's transfer share and then tax the individual to collect those shares. The problem is in deciding what each individual's transfer share (tax) ought to be.

In theory the government should use the benefit principle when determining an individual's taxes. The benefit principle is based on the fact that individuals receive some marginal benefit from the output of a given amount of public goods. We are including redistribution in the public goods category. If the government could determine the true marginal benefits that redistribution gives each individual it could tax them accordingly. With perfect knowledge the government could adjust the output of public goods in such a way that the taxes which were collected on the basis of marginal benefits were just sufficient to cover the costs of that particular bundle of public goods. In other words, the argument is that public collectivization will give us the Lindahl solution. Thus, in theory, the moving of redistribution from the private sector to the public sector could solve the free rider problem because each person could be taxed according to the marginal benefits he received from redistribution. In reality taxpayers will not reveal their true preferences and the government has no practical way of divining what these true preferences are. We move from the free rider problem of the private sector to the preference revelation problem of the public sector.

We question whether the movement of redistribution to the public sector increases the chances of obtaining a Pareto optimum. If preferences are not accurately revealed some individuals will not be taxed properly. If any individual is overtaxed he can end up with a level of utility lower than his initial utility level. If even one person is made worse off by the tax-transfer scheme of the governmental agency then we cannot automatically say that the tax-transfer scheme has led society to a preferred position. As long as an individual has an opportunity to make nontax transfers, undertaxation will not present the same type of problem.

In the voluntary situation the movement from the pre-transfer distribution to the post-transfer distribution will usually be a Pareto better move if all externalities are benevolent even when free riders are present (see Case IV). However, if some externalities are malevolent then we cannot be sure that the redistribution of goods leads to a Pareto better situation. Recall our example of two donors, one with benevolent externalities and one with malevolent. When the donor with the benevolent feelings makes a transfer to the recipient, the other (potential) donor is made worse off. Since the utility levels of two individuals have increased and that of one decreased, we are unable to say that the transfer has moved the three individuals to a better position.

Suppose malevolent externalities are present and transfers are collectivized in the public sector. Then compensation will be paid to those harmed by the transfer, if the government desires that society reach a Pareto better position. The payment of compensation means that all taxes collected will not go to those the taxpayers desire to help. Some will go to other potential taxpayers. Compensation will reduce the incentives individuals have for revealing their true preferences. They may understate their true desires for transfers in the hopes of receiving compensation or reducing their tax burden.

Payment of compensation also means that those who do pay taxes will pay lower taxes even if they have truthfully revealed their preferences concerning transfers. The reason for this is easy to see. With no compensation the optimal tax will be the one which sets

$$\frac{\partial U_1}{\partial X_1} - [\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_1}] = 0; \quad \text{tax} = dX_1 = -dX_3. \tag{3.56}$$

When compensation is paid $dX_1 > -dX_3$; the tax paid is greater than the transfer to the desired recipients. Thus, if the same tax as in equation 3.56 is paid:

$$\frac{\partial U_1}{\partial X_1} > -\left[\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}\right]\frac{dX_3}{dX_1}; \quad dX_1 > -dX_3. \tag{3.57}$$

The taxpayer's utility will increase if his taxes are reduced.

The public solution may be no solution if the amount of compensation claimed exceeds the amount of taxes paid. If the government does not compensate those harmed by the transfer, then reducing the free rider problem may move society to a Pareto worse position. Public collectivization is not necessarily better than private collectivization, the free rider problem not withstanding. Since we have assumed nonappropriation and since taxation is obviously a form of appropriation, we will no longer concern ourselves with the public collectivization of redistribution.

Case III: Nested donors with mutually exclusive nondonor recipients

In case III we assume that some donors are potential recipients of other donors. It is assumed that these donor-recipients are nested in such a manner that transfers between donors can flow in only one direction. In other words, in this case (and in the next) there is no possibility of reciprocal transfers between any two donors. We assume in case III that all nondonor recipients of each donor form mutually exclusive sets. Case III demands our attention since it shows that voluntary transfers can be motivated by malice as well as by kindness.

We will use a simple four person model in our analysis of case III. Individuals 1 and 2 are designated potential donors and individuals 2, 3 and 4 potential recipients. The utility functions for each individual are:

$$U_1 = U_1(X_1, X_2, X_3; U_2, U_3), \tag{3.58}$$

$$U_2 = U_2(X_2, X_4; U_4), \tag{3.59}$$

$$U_3 = U_3(X_3), \tag{3.60}$$

and

$$U_4 = U_4(X_4). \tag{3.61}$$

Their first order condition for utility maximization are:

$$dU_1 = 0 = \frac{\partial U_1}{\partial X_1} dX_1 + [\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_2}] dX_2$$

$$+ [\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}] dX_3 + [\frac{\partial U_1}{\partial U_2}(\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4}\frac{\partial U_4}{\partial X_4})] dX_4, \tag{3.62}$$

$$dU_2 = 0 = \frac{\partial U_2}{\partial X_2} dX_2 + [\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4}\frac{\partial U_4}{\partial X_4}] dX_4, \tag{3.63}$$

$$dU_3 = 0 = \frac{\partial U_3}{\partial X_3} dX_3, \tag{3.64}$$

and

$$dU_4 = 0 = \frac{\partial U_4}{\partial X_4} dX_4. \tag{3.65}$$

Individuals 3 and 4 will never make transfers since they are economically selfish. Individual 2 will transfer only to individual 4, if he makes any transfers at all. Individual 1 might make transfers to any or all of individuals 2, 3, and 4. Individual 4 does not enter directly into individual 1's utility function, but he does enter individual 2's. Since individual 1 is concerned with individual 2's welfare and individual 2 is concerned with individual 4's welfare, individual 4's welfare will indirectly enter individual 1's utility function.

A donor will make a transfer to a potential recipient if the benefits of making the transfer outweigh the costs. Individual 1's transfers to individual 3 and individual 2's transfers to individual 4 cause no problems. These transfers will follow from the procedure outlined in the first section of this chapter. What is of interest here is how individual 1's transfers to individual 2 will affect individual 2's transfers to individual 4; how individual 2's transfers to individual 4 will affect individual 1's transfers to individual 2; under what circumstances individual 1 will make a transfer directly to individual 4 without making a transfer to individual 2; and, under what circumstances individual 1 will make simultaneous transfers to both individual 2 and individual 4.

When individual 1 makes a transfer to individual 2, individual 2 will have an increased incentive for making a transfer to individual 4 assuming all externalities are benevolent. Individual 1's transfer to individual 2 will increase 2's holdings of goods. The increase in his holdings of goods will lower his marginal utility from holding goods ($\partial U_2/\partial X_2$ falls) which lowers his cost of making transfers. Since the benefits of making a transfer to individual 4 have not changed and his costs have fallen, individual 2's incentive for making the transfer will have increased. If individual 2 dislikes individual 4 then, obviously, individual 1's transfers to individual 2 will have no effect upon individual 2's transfers to individual 4.

How will individual 2's transfers to individual 4 affect individual 1's transfers to individual 2? Looking back at equation 3.62 we see that individual 1's decision to make a transfer to individual 2 will depend upon the values of $\partial U_1/\partial X_1$, the cost of making the transfer, and $[\partial U_1/\partial X_2 + \partial U_1/\partial U_2 \cdot \partial U_2/\partial X_2]$, the benefits from making the transfer. Individual 2's transfer to individual 4 will not affect $\partial U_1/\partial X_1$, but it will affect the term in brackets. The transfer by individual 2 will raise individual 2's utility which will cause $\partial U_1/\partial U_2$ to fall since we have assumed $\partial^2 U_1/\partial U_2^{\,2}$ is negative. On the other hand, the transfer has lowered individual 2's holdings of goods which will increase both $\partial U_1/\partial X_2$ and $\partial U_2/\partial X_2$. The effect on $[\partial U_1/\partial X_2 + \partial U_1/\partial U_2 \cdot \partial U_2/\partial X_2]$ is indeterminate since $\partial U_1/\partial X_2$ has increased and $\partial U_1/\partial U_2 \cdot \partial U_2/\partial X_2$ may have increased or decreased. If individual 1 is impressed with individual 2's benevolent behavior then we would probably be safe in assuming that $[\partial U_1/\partial X_1 + \partial U_1/\partial U_2 \cdot \partial U_2/\partial X_2]$ has increased as a result of individual 2's transfers. This would increase individual 1's incentive to make a transfer to individual 2.

There are two instances when individual 1 will transfer directly to individual 4 without making a transfer to individual 2. If all externalities are benevolent then individual 1 will make a transfer directly to individual 4 without transferring to individual 2 when

$$\left[\frac{\partial U_1}{\partial U_2}\left(\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4}\frac{\partial U_4}{\partial X_4}\right)\right] > \left[\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_2}\right]. \qquad (3.66)$$

Note that for this to occur individual 2 must also be willing to make a transfer to individual 4. This can be easily seen if we rewrite equation 3.66 as

$$\left[\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\left(\frac{\partial U_2}{\partial X_2} - \frac{\partial U_2}{\partial X_4} - \frac{\partial U_2}{\partial U_4}\frac{\partial U_4}{\partial X_4}\right)\right] < 0. \tag{3.67}$$

The term in parentheses within the brackets is similar to equation 3.63 when $dX_4 = -dX_2$. If the term in parentheses is equal to zero then individual 2 will be maximizing his utility given his bundle of goods and the externalities he faces. However, if all externalities are benevolent as we have assumed, the term in brackets in equation 3.67 will be greater than zero when the term in parentheses is equal to zero. This violates equation 3.67. Equation 3.67 will hold only when the term in parentheses is negative. Thus individual 1 will only make transfers to individual 4 when individual 2 is also willing to make a transfer to individual 4.

If individual 2 does make a transfer to individual 4, the term in parentheses in equation 3.67 will rapidly approach zero. This will limit individual 1's transfers to individual 4. If individual 2 decides to become a free rider, then individual 1's transfers to individual 4 will also cause the term in parentheses to approach zero. It will not reach zero, however, since the presence of the positive $\partial U_1/\partial X_2$ term means individual 1 will stop transferring exclusively to individual 4 before individual 2's desire to make a transfer to individual 4 is eliminated. (Note that no matter who transfers to individual 4, the $\partial U_1/\partial U_2$ term in equation 3.67 will fall as individual 2's utility increases.)

Individual 1 will transfer to individual 4 without transferring to individual 2 only as long as the inequality in equation 3.67 holds. When the inequality in 3.67 becomes an equality, individual 1 will make transfers to both individuals 2 and 4 until the marginal cost of such transfers is just equal to the marginal benefits. At this point individual 2 will desire to make further transfers to individual 4 since the existence of a positive $\partial U_1/\partial X_2$ in equation 3.67 implies that the term in parentheses is negative. This transfer by individual 2 may cause individual 1 to make an additional transfer to him.

Since individual 4's welfare enters individual 1's utility function indirectly we generally expect the same post-transfer distribution of goods to result if individual 1 does not make a transfer to individual 4 but rather makes his transfer to individual 2 and lets individual 2 make a transfer to individual 4. However, this holds only when individual 2's transfer does not lower individual 1's incentive to make further transfers to individual 2. That is, $[\partial U_1/\partial X_2 + \partial U_1/\partial U_2 \cdot \partial U_2/\partial X_2]$ must not fall as a result of individual 2's transfer to individual 4. If this term does fall, then individual 1 will transfer fewer units of the goods bundle when he makes several incremental transfers to individual 2 and lets individual 2 make all the transfers to individual 4.

Individual 1 might also be motivated to make a transfer to individual 4 without making a transfer to individual 2 when all externalities are malevolent. In this instance individual 1 may be able to increase his own utility by lowering that of individual 2. The assumption of nonappropriation precludes lowering individual 2's utility by taking some of his goods, but it does not prevent lowering his utility by increasing the utility levels of those he dislikes. Individual 1 will make the transfer if the benefits of the transfer exceed the costs; that is, if

$$\frac{\partial U_1}{\partial X_1} < \frac{\partial U_1}{\partial U_2} \left(\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4} \frac{\partial U_4}{\partial X_4}\right), \tag{3.68}$$

where $\partial U_1/\partial U_2$, $\partial U_2/\partial U_4$, and $\partial U_2/\partial X_4$ are all negative.

We might generally expect to find transfers motivated by malevolent feelings only when individual 1 has an extreme dislike for individual 2 or individual 2 has an extreme dislike for individual 4. Certainly hatred can precipitate such transfers, but it is not a necessary ingredient. Many harmless actions whose purpose is to tease or annoy some individual would be included as malevolently motivated transfers. An example of this type of behavior would be one adolescent confiding to his friend, "I like being nice. It makes my parents wonder what I'm up to." The motivation for many spiteful actions which are directed by one individual towards another may be explained by malevolent externalities in interdependent utility functions.

Case III has shown that transfers may be motivated by malevolent externalities. It has also shown how transfers are affected when some donors are the recipients of other donors.

Case IV: Nested donors with joint nondonor recipients

Case IV is a combination of cases II and III. It is assumed in case IV that some donors are the recipients of other donors. These donor-recipients are nested as in case III. It is also assumed that a donor and a donor-recipient may have common potential recipients. These common potential recipients may or may not be other potential donors. This case is important since it shows that the feelings of the donor-recipient will influence the size of his donor's transfers to the common recipient. This influence is a form of social pressure.

We will use a three person, two donor model to study this case. The utility functions of the three individuals are:

$$U_1 = U_1(X_1, X_2, X_3; U_2, U_3), \tag{3.69}$$

$$U_2 = U_2(X_2, X_3; U_3), \tag{3.70}$$

and

$$U_3 = U_3(X_3). \tag{3.71}$$

The first order conditions for utility maximization are:

$$dU_1 = 0 = \frac{\partial U_1}{\partial X_1} dX_1 + [\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_2}] dX_2$$

$$+ [\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3} + \frac{\partial U_1}{\partial U_2}(\frac{\partial U_2}{\partial X_3} + \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial X_3})] dX_3, \tag{3.72}$$

$$dU_2 = 0 = \frac{\partial U_2}{\partial X_2} dX_2 + [\frac{\partial U_2}{\partial X_3} + \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial X_3}] dX_3, \tag{3.73}$$

and

$$dU_3 = 0 = \frac{\partial U_3}{\partial X_3} dX_3. \tag{3.74}$$

The second order conditions for utility maximization are assumed to hold.

Before we analyze this three person model, we will generalize the influence individual 2 has on individual 1's transfers to individual 3. If both externalities concerning individual 2 are of the same type (i.e., the feelings individual 1 has for individual 2 and those individual 2 has for individual 3 are either both benevolent or both malevolent), then individual 1 will have a greater incentive to make a transfer to individual 3 than when individual 2 did not enter his utility function. If the externalities concerning individual 2 are of different types, then individual 1 will have a lesser incentive to make a transfer to individual 3.

To see this, look at Figure 3.8. The curve q_oq_2 is individual 1's Lindahl curve when individual 2 does not enter his utility function. When both externalities concerning individual 2 are either benevolent or malevolent, then individual 1's Lindahl curve will shift up to $q_o'q_2'$. When one externality concerning individual 2 is benevolent and the other is malevolent, then the Lindahl curve will shift down to $q_o''q_2''$. These shifts are caused by individual 1's concern for individual 2's feelings.

If all externalities in our three person model are malevolent, the only transfer that might be made is one from individual 1 to individual 3. If we set $dX_3 = -dX_1$ in equation 3.72 and set $dX_2 = 0$ in this same equation, then individual 1 will make a transfer to individual 3 whenever

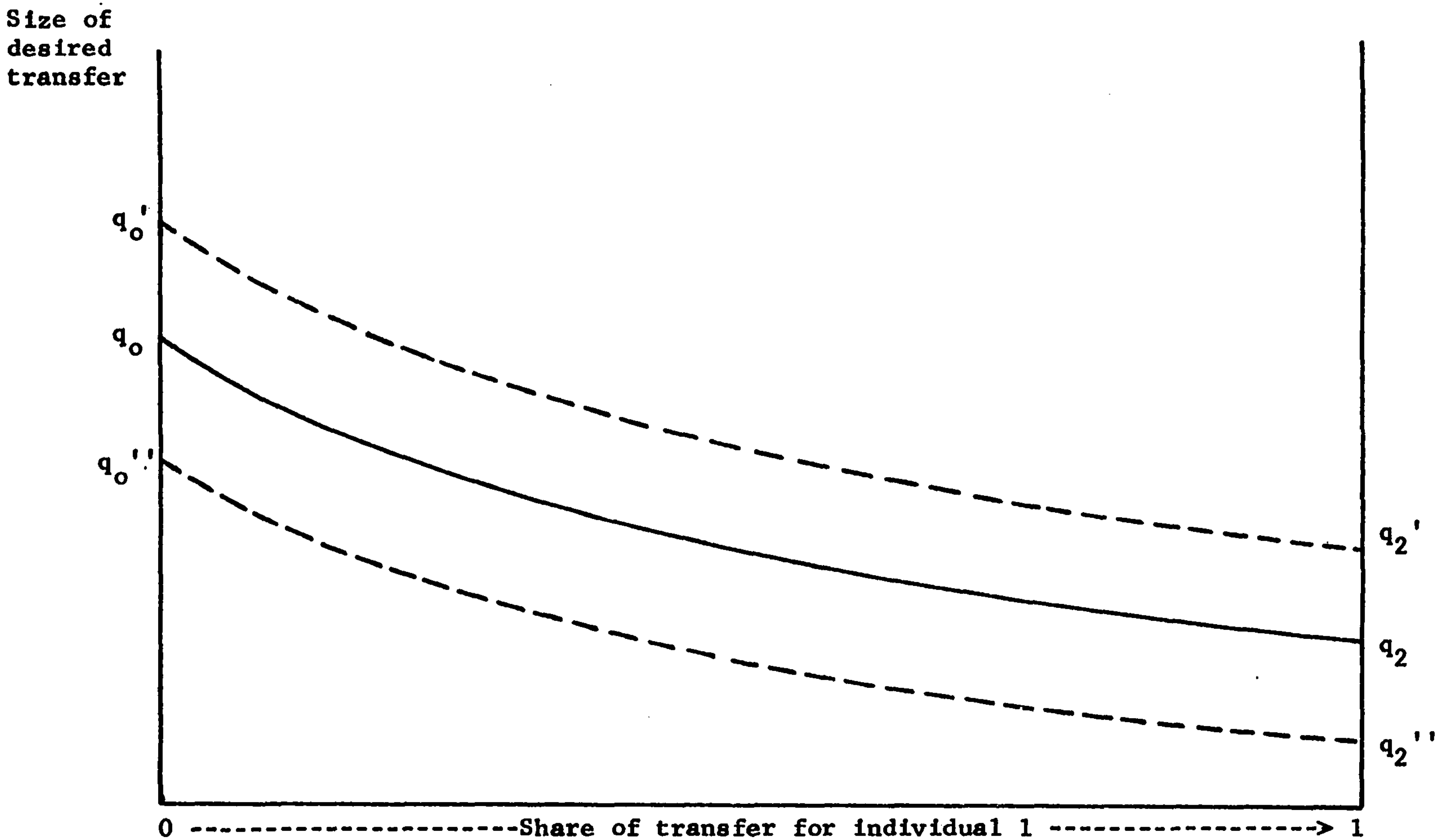

Figure 3.8. A donor's Lindahl curve when his transfers are influenced by another donor

$$\frac{\partial U_1}{\partial X_1} - \frac{\partial U_1}{\partial X_3} - \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3} - \frac{\partial U_1}{\partial U_2}\left[\frac{\partial U_2}{\partial X_3} + \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial X_3}\right] < 0. \tag{3.75}$$

The positive terms in equation 3.75 are $\partial U_1/\partial X_1$ and $\partial U_3/\partial X_3$. All other terms are negative. Individual 1 will only make the transfer when his marginal costs, comprised of his loss of utility due to his own lowered consumption and the increase in individual 3's consumption and welfare levels, are less than his marginal benefits. These marginal benefits are the increase in his utility caused by individual 2's lowered utility level. We would not expect transfers motivated by malevolent feelings to take place unless these negative feelings are extremely strong.

In terms of Figure 3.8, individual 1's Lindahl curve for a transfer to individual 3 originally coincides with the horizontal axis. "Originally" in this case means individual 2 does not exert externalities on individual 1's utility function. When individual 2 enters individual 1's utility function, the Lindahl curve may shift upward. Whether it shifts up or not will depend upon the strength of the externalities concerning individual 2 <u>vis-a-vis</u> the strength of individual 1's negative feelings for individual 3. That is, the absolute value of $\partial U_1/\partial U_2[\partial U_2/\partial X_3 + \partial U_2/\partial U_3 \cdot \partial U_3/\partial X_3]$ must be greater than $[\partial U_1/\partial X_3 + \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3]$ if individual 1's Lindahl curve is to shift upward. The horizontal axis is individual 2's Lindahl curve.

Suppose that individual 1 likes individual 3 but dislikes individual 2 and that individual 2 dislikes individual 3. In this situation individual 1's incentive for making a transfer to individual 3 will be greater than it would be if individual 1 did not care about individual

2's feelings. The positive terms in equation 3.75 are now $\partial U_1/\partial X_1$, $\partial U_1/\partial X_3$, $\partial U_1/\partial U_3$, and $\partial U_3/\partial X_3$. Individual 1 will receive an increase in his utility both from the increases in individual 3's utility level and goods holdings and from the decrease in individual 2's utility level. In this case individual 1's "original" Lindahl curve is above at least a part of the horizontal axis. The malevolent externalities in those terms concerning individual 2 will shift the Lindahl curve upward. There is no guarantee that this new, higher Lindahl curve will insure individual 1's willingness to make a unilateral transfer to individual 3.

If both donors have benevolent feelings for individual 3 but individual 1 does not like individual 2, we have a situation similar to that presented in case II. The fact that individual 2 receives an increase in his utility level when individual 3 becomes better off will dampen the utility individual 1 receives from individual 3's good fortune. Individual 1 might still make a transfer to individual 3 but he will not transfer as much as he would if he did not dislike individual 2. In this case individual 1's Lindahl curve will shift down from its "original" position.

If individual 1 likes individual 2 but both donors dislike individual 3, then the only transfer that might take place is one from individual 1 to individual 2. Individual 1 will make the transfer if his marginal costs from doing so are less than the marginal benefits he receives. Likewise, if individual 1 dislikes both individuals 2 and 3 and individual 2 likes individual 3, then individual 1 will never make a transfer although individual 2 might make one to individual 3.

When individual 1 likes both of the other individuals but individual 2 does not like individual 3, individual 2's dislike will reduce the desire individual 1 has for making a transfer to individual 3. This does not mean individual 1 will refuse to make a transfer to individual 3, only that the transfer that is made will be smaller than it otherwise would have been. Individual 2's dislike for individual 3 will not affect individual 1's transfers to individual 2.

Suppose that individual 1 has benevolent feelings for individual 2 and malevolent feelings for individual 3 and that individual 2 has benevolent feelings for individual 3. If the externality is large enough, individual 2 will make a transfer to individual 3. Individual 1 might also make a transfer to individual 3. Look at equation 3.75. If the $\partial U_1/\partial U_2[\partial U_2/\partial X_3 + \partial U_2/\partial U_3 \cdot \partial U_3/\partial X_3]$ term is larger than $\partial U_1/\partial X_1 - \partial U_1/\partial X_3 - \partial U_1/\partial U_3 \cdot \partial U_3/\partial X_3$, then individual 1 will be willing to make a transfer to individual 3. This may explain the "I didn't really want to do it, but it meant so much to Mr. Smith" actions that we all occasionally perform. The benefits to individual 2 are so large that individual 1 will make the transfer to individual 3 even though he does not like individual 3. Note that it is not necessary that individual 2 be willing to make a transfer to individual 3 for the above argument to hold. This argument depends only upon the benefits individual 2 receives from individual 3's increased well-being and upon the benefits individual 1 receives from the increase in individual 2's utility. Although the marginal benefits to individual 2 may be very large, it is possible that his marginal cost of making a transfer to individual 3 is so great that he would not undertake such a transfer alone.

Assume that these same feelings hold. If individual 1's knowledge of individual 2's benevolent feelings for individual 3 is not enough to offset his own dislike for individual 3, then individual 1 might reduce his transfers to individual 2 when individual 2 transfers goods to individual 3. The reason individual 1 might reduce his transfer is that individual 2's transfer to individual 3 can cause a reduction in individual 1's utility. Individual 2's transfer will reduce his own holdings of goods and increase individual 3's holdings and utility level. Both of these actions will lower individual 1's utility level. This transfer will increase individual 2's utility level, however, which will increase individual 1's utility level. The total effect on individual 1's utility level is indeterminate. If individual 1's utility level decreases, he will reduce his transfers to individual 2 since that portion of his transfer which is retransferred by individual 2 imposes an additional cost on him.

Finally, consider the situation in which all externalities are benevolent. This is basically the situation Hochman and Rodgers [22] had in mind when they wrote their article on Pareto optimal redistribution. They did not seem to consider individual 2's influence on individual 1's decision of whether or how much to transfer to individual 3. Perhaps they thought this effect was negligible. It is possible in this situation for individual 1 to make transfers to both individuals 2 and 3 and for individual 2 to make a transfer to individual 3.

Individual 1's transfer to individual 2 will increase the incentive individual 2 has for making a transfer to individual 3 since his marginal cost for making the transfer will be lowered. Likewise, individual 2's concern for individual 3's welfare will increase the incentive individual 1 has for making a transfer to individual 3.

Can individual 1 ever be a free rider in this situation? Not if he transfers goods to individual 2 and individual 2 transfers goods to individual 3. Individual 2 will pass some of these transfers on to individual 3. In this case individual 1 will make his transfer to individual 3 indirectly, but he will share in the transfers nonetheless. If individual 1 does not make a transfer to individual 2 or individual 3, then he can be a free rider.

Individual 2 can be a free rider but any transfers he receives will tend to reduce his desire to free ride. As we stated earlier, an individual will not be a free rider unless he receives a greater increase in his utility from free riding than he does from making a joint transfer.

Case IV has one major offering. It is that an individual's actions can be influenced by the desires of other individuals. An individual may be willing to undertake actions he would rather not perform if he knows these actions will please some other person. No use of threat or force need be made in these instances. Likewise, the individual might limit some activity he enjoys because some person he cares for does not approve. Both types of influence are forms of social pressure.

<u>Case V: Reciprocal interdependencies with mutually exclusive nondonor recipients</u>

In case V we address the problem of reciprocal interdependencies. We We will utilize a four person model in which both donors are potential recipients of the other donor. The nondonor recipients are assumed to form mutually exclusive sets as in cases I and III. The utility functions of the four individuals are:

$$U_1 = U_1(X_1, X_2, X_3; U_2, U_3), \tag{3.76}$$

$$U_2 = U_2(X_1, X_2, X_4; U_1, U_4), \tag{3.77}$$

$$U_3 = U_3(X_3), \tag{3.78}$$

and

$$U_4 = U_4(X_4). \tag{3.79}$$

The first order conditions for utility maximums are:

$$\begin{aligned} dU_1 = {} & \frac{1}{1 - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}} \Big[\Big(\frac{\partial U_1}{\partial X_1} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_1}\Big)\, dX_1 \\ & + \Big(\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_2}\Big)\, dX_2 + \Big(\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}\Big)\, dX_3 \\ & + \frac{\partial U_1}{\partial U_2}\Big(\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4}\frac{\partial U_4}{\partial X_4}\Big)\, dX_4 \Big] = 0, \end{aligned} \tag{3.80}$$

$$\begin{aligned} dU_2 = {} & \frac{1}{1 - \frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial U_2}} \Big[\Big(\frac{\partial U_2}{\partial X_1} + \frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial X_1}\Big)\, dX_1 \\ & + \Big(\frac{\partial U_2}{\partial X_2} + \frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial X_2}\Big)\, dX_2 + \frac{\partial U_2}{\partial U_1}\Big(\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}\Big)\, dX_3 \end{aligned}$$

$$+ (\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4} \frac{\partial U_4}{\partial X_4}) \, dX_4] = 0, \tag{3.81}$$

$$dU_3 = \frac{\partial U_3}{\partial X_3} \, dX_3 = 0, \tag{3.82}$$

and

$$dU_4 = \frac{\partial U_4}{\partial X_4} \, dX_4 = 0. \tag{3.83}$$

The denominator of equations 3.80 and 3.81 is $1 - \partial U_1/\partial U_2 \cdot \partial U_2/\partial U_1$ instead of 1 as was previously the case since each donor now enters the other's utility function. In earlier cases $\partial U_2/\partial U_1$ was equal to zero. Each donor's marginal benefit from own consumption has also changed since he must now consider how his own increase or decrease in consumption will affect the other donor's utility level.

There are ten different combinations of malevolent and benevolent externalities for case V. We will cover most of these situations rapidly since they are fairly similar to situations discussed in cases II through IV. The discussion will be more detailed for those cases where each donor has some incentive to transfer to the other.

When all externalities are malevolent, the transfers which might occur are those from individual 1 to individual 4 and from individual 2 to individual 3. These spiteful transfers are made for the purpose of lowering the other donor's level of utility and, hence, raising one's own. If both donor's make this type of transfer, it is possible that they will both end up worse off. Neither would make the transfer unless it raised his own utility level. However, the loss in utility each donor experiences from the other's transfer may more than offset the increase he receives from making his own transfer.

If one donor has benevolent feelings for the nondonor individual who enters his utility function and if all other externalities are malevolent, then the donor with the benevolent feelings will transfer to both of the nondonors if such transfer will increase his utility. He will make a transfer to one nondonor because he likes him and to the other because it lowers the other potential donor's utility level.

If each donor dislikes the other but likes the nondonor entering his utility function, then each will make a transfer to the proper nondonor if the externality is strong enough. He will not make a transfer to the other nondonor.

When one donor likes the other and all other externalities are malevolent, then the donor with the benevolent externality may make a transfer to the other. The donor-recipient may refuse to accept the offered transfer, however, since he may be made worse off by accepting. The donor-recipient's utility will be increased, if he accepts the transfer, by the fact that he will have more goods and the donor will have fewer goods. It will be lowered by the donor's increased utility. The total effect on the utility level of the donor-recipient is indeterminate. If the transfer increases his utility level, he will accept it. If not, he will refuse it.

The strength of the donor-recipient's malevolent feelings toward the donor will determine whether he is the sort who would say "I don't like him, but if he is going to give money away I'll be the first one in line" or "I wouldn't accept anything from him; not even if I were starving and he offered me the last apple on earth."

This same situation may find the donor-recipient making a transfer to the individual who is disliked by the donor. This transfer may take place whether or not the donor-recipient receives a transfer from the donor.

If one donor is purely altruistic and likes both of the other individuals entering his utility function and the other donor dislikes both individuals entering his utility function, then the first donor will try to make transfers to each of the two individuals who enter his utility function, given appropriate levels of the externalities. The second donor would not make any transfer. As in the previous situation, he has the option of refusing the transfer offered him.

Suppose that one donor, say individual 1, likes individual 2 but dislikes individual 3. Suppose also that individual 2 likes individual 4 and dislikes individual 1. In this situation, individual 1 might offer a transfer to individual 2. Individual 2 will accept or refuse the offer depending upon whether it would increase or decrease his utility level. Individual 1 might also make a transfer to individual 4, but, as in case III, he will only offer the transfer if individual 2 is also willing to make a transfer to individual 4. Individual 2 might make a malicious transfer to individual 3 and a benevolent transfer to individual 4.

The situation in which all externalities are benevolent except those individual 2 has for individual 1 will lead to the same results as in the preceding situation with the exceptions that individual 1 might desire to make a transfer to individual 3 and individual 2 would not. These last four situations in which one donor, say individual 1, likes individual 2 but individual 2 does not like individual 1 remind us of a parent who is having problems relating to his teenaged child. The child (individual 2) never seems to appreciate the sacrifices the parent (individual 1) makes for him. As a matter of fact the child will occasionally, and sometimes persistently, do things which upset the parent. I.e., he will make a transfer to individual 3 when individual 1 dislikes individual 3.

In the next situation let both donors have benevolent feelings for each other but malevolent feelings for the nondonors. Since neither donor will make a transfer to a nondonor the situation is essentially the same as that presented at the end of the third section of Chapter II. If the benevolent externalities are not great enough for either donor to make a transfer to the other, then the donors will be in a region of pure exchange. In the two good case this will be region 1 of Figure 2.13 which is reproduced here in a modified form as Figure 3.9.

If only one donor has externalities strong enough to cause a transfer, then he will offer the other donor all or some of the goods in his consumption bundle. If individual 1 is the donor, he will be in region 2 of Figure 3.9 if he desires to transfer some of each good in his consumption bundle. He will be in region 4 or region 6 if he desires to transfer only one good in this two good example. Since individual 2's benevolent externality is not strong enough to cause him to transfer to individual 1 he will accept the transfer offered by individual 1. As he accepts more units of the transfer his marginal benefits from the

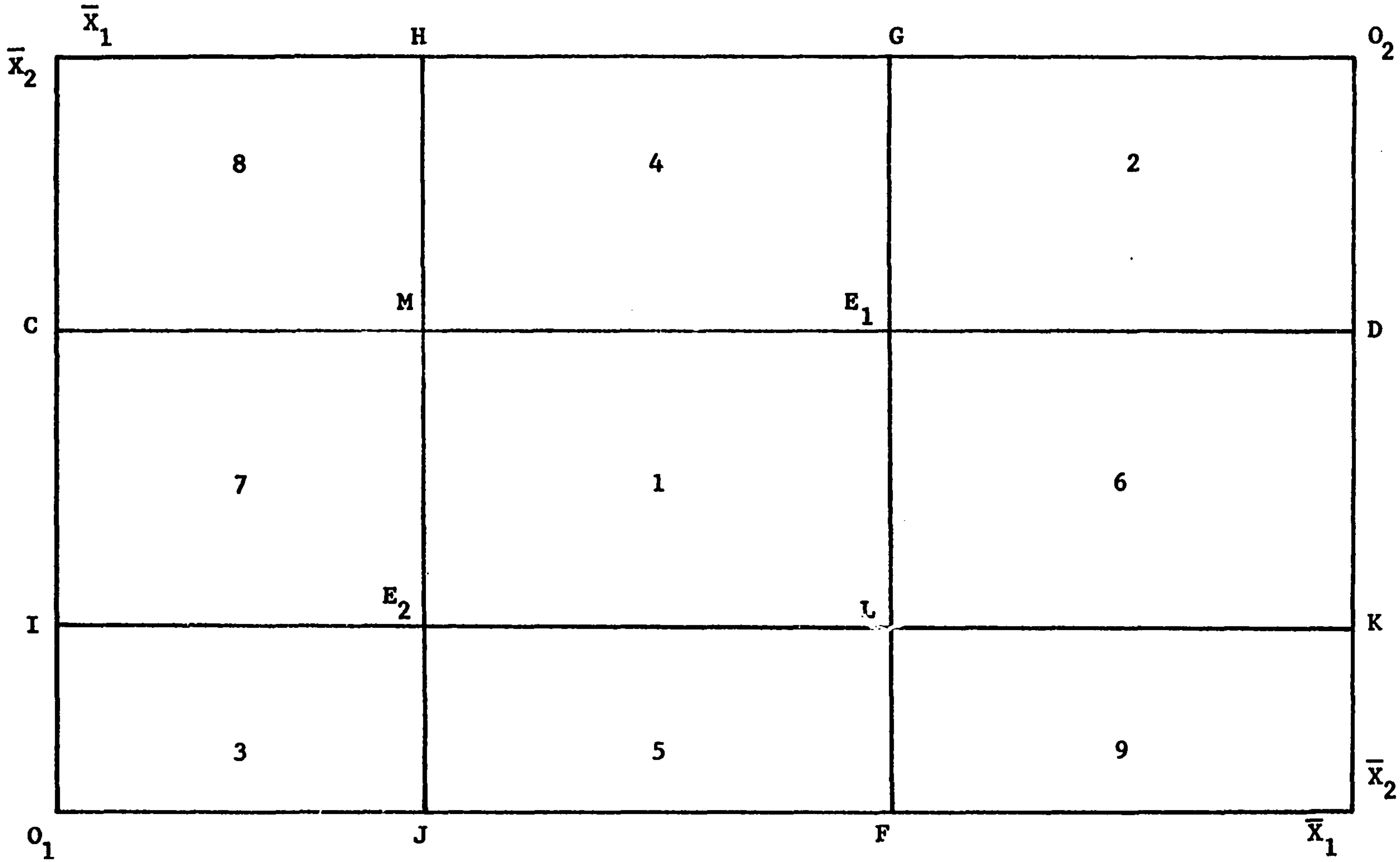

Figure 3.9. Edgeworth box when both donors are benevolent

transfer will fall. His marginal costs $(\partial U_2/\partial X_1 + \partial U_2/\partial U_1 \cdot \partial U_1/\partial X_1)$ may rise or fall. Both $\partial U_2/\partial X_1$ and $\partial U_1/\partial X_1$ will rise as individual 1 transfers goods to individual 2. However, $\partial U_2/\partial U_1$ will fall since the transfer makes individual 1 better off. The interaction of these three terms may lead to a rise or a fall in individual 2's marginal cost from receiving the transfer. The transfer from individual 1 to individual 2 will stop either when individual 1's marginal cost from making the transfer is just equal to the marginal benefits he receives or when the marginal benefits individual 2 obtains from the receipt of the transfer is just equal to the marginal cost he incurs. Either individual may stop the transfer.

Suppose each donor desires to make a transfer to the other. Can reciprocal transfers take place? Would one individual be willing to accept a transfer from an individual to whom he desired to make a transfer? The answer to these questions is dependent upon the particular transfers each donor desires to make. We have reproduced Figure 2.14 as Figure 3.10. Recall that in this figure each individual's bliss point is closer to his own origin than it is to the other individual's origin.

If both donors want to transfer some of each good in their consumption set then the initial allocation would lie in region 1 of Figure 3.10. As we stated in Chapter II this region is a contract region. All allocations within this region are Pareto efficient. Each individual would refuse to accept a transfer offered by the other since accepting the transfer would move him away from his own bliss point. If the

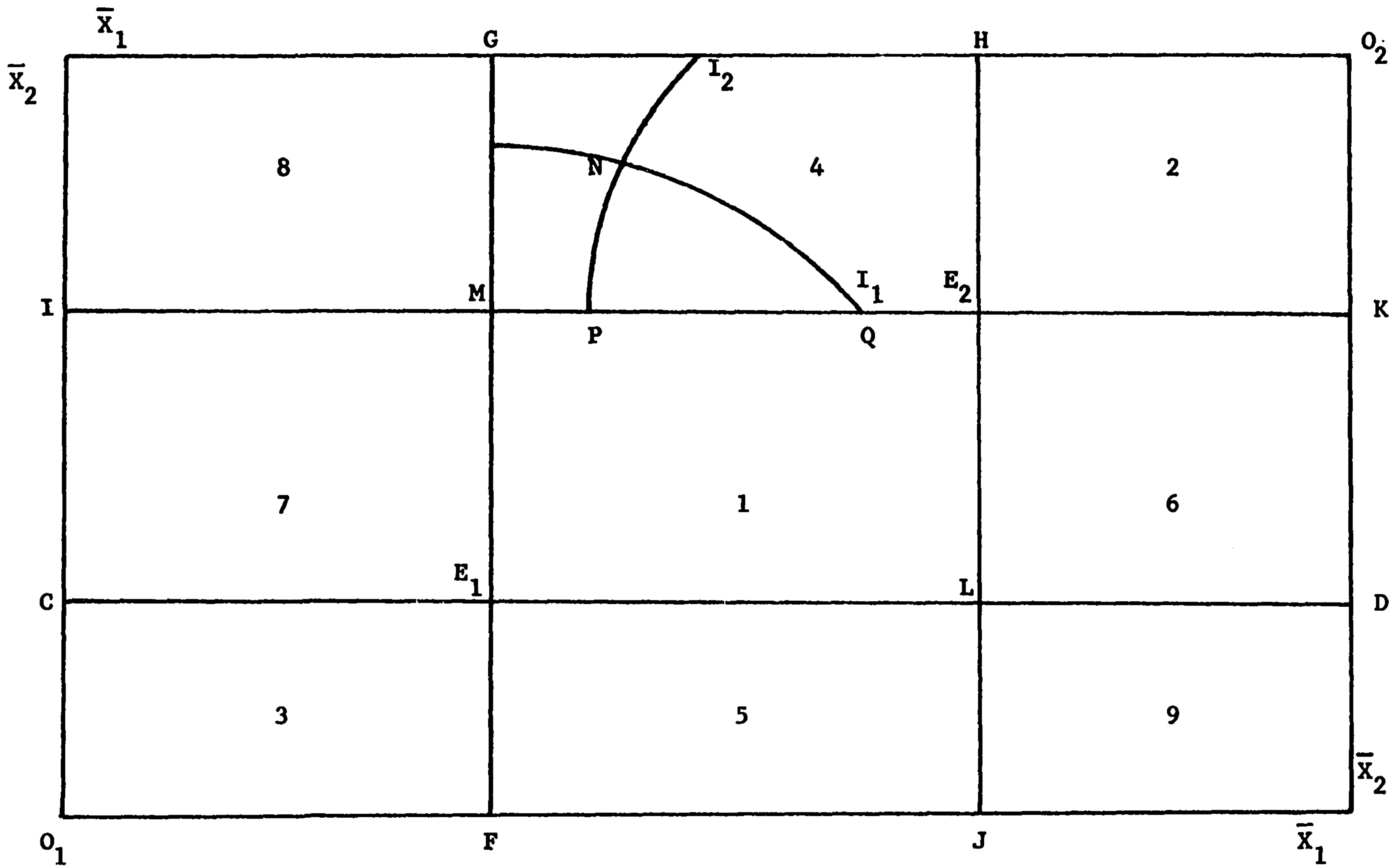

Figure 3.10. Edgeworth box when both donors are very benevolent

custom of his society forbade him from refusing the transfer, he would accept it, wait an appropriate amount of time, and make his own transfer to the original donor. This process would be repeated by the original donor, and then, again, by the original recipient. There would not be a stable equilibrium in this society. Transfers would continuously move the equilibrium from one individual's bliss point to the other's.

If the desired transfers are such that one donor wants to make a transfer of a subset of the goods bundle and the other donor wants to make a transfer of a different subset so that the two transfer bundles do not have any goods in common, then the transfers will be made without being rejected. In the two good case, this type of transfer will occur if the initial allocation is in region 8 or 9 of either Figure 3.9 or 3.10. In region 8 of either figure, individual 1 desires to transfer some of good X_2 to individual 2 and individual 2 desires to transfer some of good X_1 to individual 1. They will continue their transfers, which resemble exchanges since each is giving a good to the other, until point M is reached. In region 9 each will transfer the other good until point L is reached.

Suppose each donor's transfer bundle contains some goods in common with the other donor's transfer bundle. This is the situation that occurs in regions 4, 5, 6, and 7 of Figure 3.10. In the two person, two good case depicted, one individual wants to transfer some of each good and the other wants to transfer some units of one good. The goods which are not common to both transfer bundles will be transferred without any problems. Whether the goods common to both transfer bundles are transferred and who receives them if they are transferred will depend upon the relative persistence and insistence capabilities of the two donors.

In region 4 of Figure 3.10 we have drawn portions of the two individuals' indifference curves, I_1 and I_2, which contain the initial distribution, point N. At point N, individual 1 wants to transfer some of both goods while individual 2 wants to transfer some of good X_1. Individual 1 will transfer enough of good X_2 to individual 2 that the ridge line $IMPQE_2K$ will be reached. The final post-transfer distribution will lie somewhere along the PQ portion of this ridge line. It's exact location will depend upon the bargaining powers of the two individuals.

The last situation we will discuss in case V is one in which all externalities are benevolent. Each donor will make a transfer to his "own" nondonor recipient if the externalities are strong enough to warrant such a transfer. A donor's transfer to his "own" nondonor recipient will have an effect upon the other donor's desire to transfer to him. If we assume individual 1 makes a transfer to individual 3, we can look at equation 3.81 to find what effect this transfer will have on individual 2. Letting $-dX_1 = dX_3$, $dX_2 = dX_4 = 0$, and $1/1-\partial U_1/\partial U_2 \cdot \partial U_2/\partial U_1 = A$ we can rewrite equation 3.81 as

$$dU_2 = A\left[\frac{\partial U_2}{\partial U_1}\left(\frac{\partial U_1}{\partial U_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3} - \frac{\partial U_1}{\partial X_1}\right) - \frac{\partial U_2}{\partial X_1}\right] dX_3. \tag{3.84}$$

Initially individual 1's transfers to individual 3 might increase individual 2's utility level. Since all of the partial derivatives within the brackets are positive, individual 1's transfers will eventually cause individual 2's utility level to decline. Individual 1 will transfer goods to individual 3 until the term in parentheses within the brackets is equal to zero. At this point individual 2 is losing utility since $\partial U_2/\partial X_1$ is greater than zero. We know that individual 1's transfers to individual 3 will cause individual 2's utility to fall over some range of the transfer. It may also cause individual 2's utility to increase during the early stages of the transfer. The transfer might increase individual 2's utility level, but then it might also reduce it. Even though all externalities are benevolent, it is still possible for an individual to be made worse off by a voluntary transfer.

There is an example which we have heard reasonably frequently which seems to fit this situation. It is of one individual, individual 2, telling another, "I like Joe, too, and I realize that he has been having problems lately, but you did not have to be so generous." Individuals who accuse others of being generous to a fault probably have interdependent utility functions similar to that possessed by our individual 2.

Looking back at equation 3.81 we see that individual 1's transfer to individual 3 will cause $\partial U_1/\partial X_1$ and $\partial U_2/\partial X_1$ to rise. It will cause $\partial U_2/\partial U_1$ to fall. Thus the marginal benefit individual 2 will receive from making a transfer to individual 1 can increase or decrease as a result of individual 1's transfer. Individual 2's marginal cost of making a transfer (any transfer) will fall since $\partial U_2/\partial X_2$ and $\partial U_1/\partial X_2$ have not changed and $\partial U_2/\partial U_1$ has fallen. The rationale for this decrease in the marginal cost is that individual 2's marginal utility from consumption is based upon his own marginal valuation of that consumption and upon the marginal effects he feels this consumption has on individual 1. As individual 1 becomes better off, individual 2 will be less concerned with how his own consumption affects individual 1.

Individual 1's transfer to individual 3 will increase the incentive individual 2 has for making a transfer to individual 4 since individual 1's transfer will lower the marginal costs of making a transfer for individual 2. The effect individual 1's transfer has on individual 2's desire to transfer to individual 1 is not known since we do not know for sure how this transfer will affect the marginal benefits individual 2 would receive from making the transfer to individual 1.

When all externalities are benevolent the criteria for reciprocal transfers between the two donors will be the same as those described in the previous situation where the donors had benevolent feelings for each other but malevolent feelings for the nondonor recipients.

Case VI: Reciprocal interdependencies with joint nondonor recipients

Case VI is very similar to case V. The only difference is that there is now a joint nondonor recipient. The three person model for this case is:

$$U_1 = U_1(X_1, X_2, X_3; U_2, U_3), \tag{3.85}$$

$$U_2 = U_2(X_1, X_2, X_3; U_1, U_3), \tag{3.86}$$

and

$$U_3 = U_3(X_3). \tag{3.87}$$

The first order conditions for utility maximums are:

$$dU_1 = \frac{1}{1 - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}} \left[\left(\frac{\partial U_1}{\partial X_1} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_1}\right) dX_1 + \left(\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_2}\right) dX_2\right.$$

$$\left. + \left(\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3} + \frac{\partial U_1}{\partial U_2}\left(\frac{\partial U_2}{\partial X_3} + \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial X_3}\right)\right) dX_3\right] = 0, \tag{3.88}$$

$$dU_2 = \frac{1}{1 - \frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial U_2}} \left[\left(\frac{\partial U_2}{\partial X_1} + \frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial X_1}\right) dX_1 + \left(\frac{\partial U_2}{\partial X_2} + \frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial X_2}\right) dX_2\right.$$

$$\left. + \left(\frac{\partial U_2}{\partial X_3} + \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial X_3} + \frac{\partial U_2}{\partial U_1}\left(\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}\right)\right) dX_3\right] = 0, \tag{3.89}$$

and

$$dU_3 = \frac{\partial U_3}{\partial X_3} = 0. \tag{3.90}$$

There are again ten combinations of benevolent and malevolent externalities. We will not analyze this case since the results follow directly from case V. We will generalize the results, however.

Reciprocal transfers between the donors will take place under the same conditions as those discussed in case V. The donors' transfer patterns to the nondonor recipient can be obtained from case V as follows. If a donor had some incentive to transfer to both the nondonor recipients in case V, then, in case VI, his incentive to transfer to the nondonor recipient is essentially the combination of these other incentives. As in case IV, the Lindahl curve of this donor is raised from its "original" position when the externalities this donor has for the other donor and those the other donor has for the nondonor recipient are either both benevolent or both malevolent. "Original" in this instance again means the effects caused by the other donor do not enter the analysis.

If the donor had an incentive to transfer to only one of the two nondonor recipients in case V, then he would probably still have an incentive to transfer to the nondonor recipient in case VI. In this case his Lindahl curve will be lower than his "original" Lindahl curve. Finally if the donor did not desire to make a transfer to either nondonor recipient in case V, then he would have no incentive to make a transfer to the nondonor recipient in this case.

Case VII: Everyone has an interdependent utility function

The case in which everyone has an interdependent utility function is obviously the general case from which the other, more specific six cases have been obtained. The mathematics of the N-person model become unwieldly when N is greater than three so we will look at the three person model. The utility functions of our three individuals are:

$$U_1 = U_1(X_1, X_2, X_3; U_2, U_3), \tag{3.91}$$

$$U_2 = U_2(X_1, X_2, X_3; U_1, U_3), \tag{3.92}$$

and

$$U_3 = U_3(X_1, X_2, X_3; U_1, U_2). \tag{3.93}$$

The first order conditions for utility maximums are:

$$dU_i = \frac{1}{X} [A_i \; dX_1 + B_i \; dX_2 + C_i \; dX_3] = 0 \tag{3.94}$$

where

$$X = 1 - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial U_1} - \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial U_2}\frac{\partial U_2}{\partial U_1} - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1} - \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial U_1}$$

$$- \frac{\partial U_2}{\partial U_3}\frac{\partial U_3}{\partial U_2}\,, \tag{3.95}$$

$$A_i = \left(1 - \frac{\partial U_i}{\partial U_k}\frac{\partial U_k}{\partial U_j}\right)\frac{\partial U_i}{\partial X_1} + \left(\frac{\partial U_i}{\partial U_j} + \frac{\partial U_i}{\partial U_k}\frac{\partial U_k}{\partial U_j}\right)\frac{\partial U_j}{\partial X_1}$$

$$+ \left(\frac{\partial U_i}{\partial U_k} + \frac{\partial U_i}{\partial U_j}\frac{\partial U_j}{\partial U_k}\right)\frac{\partial U_k}{\partial X_1}\,, \tag{3.96}$$

$$B_i = \left(1 - \frac{\partial U_i}{\partial U_k}\frac{\partial U_k}{\partial U_j}\right)\frac{\partial U_i}{\partial X_2} + \left(\frac{\partial U_i}{\partial U_j} + \frac{\partial U_i}{\partial U_k}\frac{\partial U_k}{\partial U_j}\right)\frac{\partial U_j}{\partial X_2}$$

$$+ \left(\frac{\partial U_i}{\partial U_k} + \frac{\partial U_i}{\partial U_j}\frac{\partial U_j}{\partial U_k}\right)\frac{\partial U_k}{\partial X_2}\,, \tag{3.97}$$

and

$$C_i = \left(1 - \frac{\partial U_i}{\partial U_k}\frac{\partial U_k}{\partial U_j}\right)\frac{\partial U_i}{\partial X_3} + \left(\frac{\partial U_i}{\partial U_j} + \frac{\partial U_i}{\partial U_k}\frac{\partial U_k}{\partial U_j}\right)\frac{\partial U_j}{\partial X_3}$$

$$+ \left(\frac{\partial U_i}{\partial U_k} + \frac{\partial U_i}{\partial U_j}\frac{\partial U_j}{\partial U_k}\right)\frac{\partial U_k}{\partial X_3}\,, \tag{3.98}$$

$i = 1, 2, 3;\ i \neq j \neq k.$

Each individual's marginal valuation of his own consumption and of the consumption of the other individuals will be influenced by the feelings each individual has for each of the other individuals. Each time one individual makes a transfer to another, the marginal valuation of each person in the society will change. Each individual will change his marginal valuation not only for the goods bundles held by the individuals involved in the transfer but for the goods bundles held by every member of the society.

Thus if individual 1 makes a transfer to individual 2, not only will individual 1's marginal valuation of his own consumption and of individual 2's consumption change, but his marginal valuation of individual 3's consumption will also change. Look at equation 3.98. Let $i = 1$, $j = 2$, and $k = 3$. The transfer from individual 1 to individual 2 will increase individual 2's utility if it is accepted. This increase in individual 2's utility will cause both $\partial U_1/\partial U_2$ and $\partial U_3/\partial U_2$ to fall. Since both $\partial U_1/\partial U_2$ and $\partial U_3/\partial U_2$ are in two of the three terms on the right hand side of equation 3.98, the value of C_1 must be affected by the transfer. Any transfer between any two individuals will affect all the marginal valuations of all individuals.

If there are many individuals in the society, then we would expect most of the marginal valuations to be very small. If individual 1 and individual 3 were not acquainted with each other, then we would expect $\partial U_1/\partial X_3$, $\partial U_1/\partial U_3$, $\partial U_3/\partial X_1$, $\partial U_3/\partial U_1$ and most terms involving these four terms to be close to zero.

In our three person example we will assume that each person is well acquainted with the other two. Transfers can be motivated by benevolent or malevolent feelings as in previous cases. If transfers are motivated by benevolent feelings, the donor desires to increase the utility of the person receiving the transfer. If transfers are motivated by malevolent feelings, then the donor is not as interested in increasing the utility of the recipient as he is in decreasing the utility of some third party.

Of course it is possible that a transfer is motivated by both types of feelings. The donor wants to increase the utility of the recipient and he also wants to decrease the utility of some third party.

An individual's desire to make a transfer will be influenced by the other members in the society. The individual might make a transfer to an individual he dislikes if this transfer either pleases some individuals he likes very much (or wants to impress) or displeases some individuals he dislikes more than the recipient. In either case, the individual is still making the transfer voluntarily.

Likewise, the individual might not make a transfer that he would desire to make in the absence of the nonrecipient individuals if this transfer makes some individuals he cares for worse off or if it makes some individuals he dislikes better off.

A new development in this case is that not only can an individual's desire to make a transfer be influenced by other individuals, but his desire to accept a transfer can also be influenced by other individuals. An individual might reject a transfer if he knows that accepting it will make someone he likes worse off. Thus a child might forego a free ice cream cone if he knows that it will ruin his appetite and that his mother has put a lot of effort into preparing his dinner. The individual might also reject a transfer if accepting it increases the utility of someone he dislikes. When we hear someone say, "Of course I want (to accept) it, but I want it because I want it, not because Mr. Smith wants me to have it," we are hearing him reject a transfer because the transfer increases the utility of someone whose utility he does not want increased.

An individual might also accept a transfer he does not want because this action either increases the utility of someone he likes or reduces the utility of someone he dislikes. An example of the first reason might be an individual who accepts a gift he knows the giver cannot afford to make. He reluctantly accepts the gift because he knows the giver will receive an increase in utility from the transfer. An example of the second reason might be a girl accepting a date with a boy she does not really like in order to make her boyfriend jealous.

Reciprocal transfers will take place under the conditions outlined in case V. These conditions are if each donor wants to transfer goods not being transferred by his recipient then the transfers will take place. If the transfer bundle of each donor contains some goods in common with the transfer bundles of other donors, then those goods which are not in common will be transferred. Those goods which are in common will not be transferred unless one donor is very persistent or the other very weak-willed. When every transfer bundle contains the same goods, no one will accept the offered bundle since it would make him worse off.

The Formation of Charitable Organizations

When one individual makes a voluntary transfer to another it may be said that he is making a charitable contribution to the recipient. When two or more individuals join forces to make a joint transfer to one recipient, then they have effectively formed a charitable organization. The organization may be formally organized and have an effective life of several decades such as the Red Cross or the American Cancer Society, or it may be informal and have an effective life of only a few hours such as the "Let's give Shorty, our janitor, a Christmas present" society. The recipient of the charitable organization's transfers may be one individual, such as Shorty, or it may be an entire class of individuals such as the blind, cancer victims, or the indigent. The common thread in all charitable organizations is that each member of the organization makes a contribution to the total transfer.

We discussed an individual's desire to make a joint transfer in some detail in case II of the preceding section of this chapter. The conclusions of that case were basically that an individual would join in making a joint transfer if that gave him a higher level of utility than he obtained by being a free rider. We also noted that the smaller an individual's share of the total transfer the more likely he is to join in making that transfer. This generalization does not hold in all cases, of course, for some individuals will never willingly join in a transfer no matter how small their share.

If the recipient of the transfer is one individual or a group whose members do not change, then we may treat the recipient as we did individual 3 in case IV of the preceding section. That is, the individual who is the object of the transfer is the one who receives the transfer. We assume that this recipient keeps and consumes the goods transferred to him.

If the individuals who are the object of the transfer belong to a group whose members are constantly changing or personally unacquainted with the potential donors, then the potential donors might make their transfers to an intermediary which claims to have some knowledge of who the desired transfer recipients are. These transfers to the intermediary are made because they reduce the information costs to the donors. This situation is similar to case III of the preceding section. The donor of the transfer corresponds to individual 1 in this case; the intermediary, to individual 2; and the final recipient, to individual 4. In this case the intermediary can be one individual such as Reverend Weems, the pastor of the donor's church, or an organization such as the American Foundation for the Blind. The final recipient can be one individual such as the poorest individual in Reverend Weems' parish or it could be a group of individuals such as all blind people. We will look at this situation in more detail.

Let us rewrite equation 3.62.

$$dU_1 = 0 = \frac{\partial U_1}{\partial X_1} dX_1 + \left(\frac{\partial U_1}{\partial X_2} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial X_2}\right) dX_2 + \left(\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}\right) dX_3$$

$$+ \left(\frac{\partial U_1}{\partial U_2}\left(\frac{\partial U_2}{\partial X_4} + \frac{\partial U_2}{\partial U_4}\frac{\partial U_4}{X_4}\right)\right) dX_4. \qquad \text{(3.62) repeated}$$

This is individual 1's first order condition for a utility maximization from case III. Strictly speaking, if individual 1 desires to make a transfer to a certain class of individuals then this class of individuals should enter his utility function. If we assume that he is ignorant of exactly who the members of this class are and that individual 2, the intermediary, knows their identities, then we can assume that only individual 2 enters directly in individual 1's utility function. We must realize now, however, that when individual 1 makes a transfer to individual 2 his purpose is not to make individual 2 better off but to give individual 2 the means of making transfers to individual 4.

Ideally, individual 1 would prefer that individual 2 pass all of the transfer on to the final recipient(s). As we have seen, however, individual 2 will pass on only enough of the transfer to equate his marginal cost of the transfer with his marginal benefit. Individual 2 will almost always keep a portion of individual 1's transfer for himself. That portion of the transfer retained by the intermediary has been called fees, administrative costs, fund raising costs, profit, and graft [31]. When the donor realizes what portion of his transfer is retained by the intermediary, his transfers will be of a size that maximizes his utility. When the donor does not realize that the intermediary is retaining a portion of his transfer, or does not know what portion is being retained, then he will make the wrong size transfer. If he underestimates the intermediary's "cut," his transfers will be too large [31]. He should reduce his transfers in order to maximize his utility.

Current tax policy in the United States is designed to increase the incentive individuals have for making charitable contributions to certain charitable intermediaries or charity recipients by giving these intermediaries and recipients a tax deductible status. This tax deductible status means any charitable contribution an individual makes to one of these intermediaries or recipients may be subtracted from his income before his taxes are determined. He will pay taxes on a smaller income and the marginal tax rate he faces may be smaller as a result. Since all charitable intermediaries and recipients do not have the tax deductible status, the government is able to increase the incentive an individual has of making a transfer to an intermediary or recipient with the tax deductible status by effectively making a joint transfer with this individual.

Current federal income tax policy increases the incentive of all taxpayers to make charitable contributions, but it generally increases the incentive of individuals in high tax brackets more than the incentive of individuals in low tax brackets. We would usually expect individuals with higher incomes to make more transfers than individuals with lower incomes since we would expect their marginal utility from own consumption to be lower. If the marginal utility of own consumption falls after the basic necessities of life have been obtained, then the marginal cost of making a transfer will also fall. Even if we assume that the marginal cost of making a transfer is lower for a high income individual than for a low income individual, we will not always observe higher income individuals making larger transfers. The decision to make transfers is based upon the marginal benefits the transfer produces as well as the marginal costs.

We will use a simple example to show that the current tax policy induces larger transfers from high income individuals than from low income individuals. Suppose that there are two individuals. Individual 1 has an income of $12,000 a year and individual 2 has an income of $50,000 per year. Suppose an individual can contribute up to fifty percent of his income to a charitable cause before he has to pay taxes on those charitable contributions. An individual's income tax is based on his income after deductions for charitable contributions. We will assume that both of our individuals contribute fifty percent of their income to charitable causes.

If the average tax rate on a \$12,000 income is twenty percent, our first individual would owe \$2400 in taxes if he did not make any charitable contributions. We assume that he transfers \$6,000 to some worthy cause so he need pay taxes on only \$6,000. Assume the average tax rate on \$6,000 is fifteen percent. Then the first individual owes \$900 in taxes. Our first individual has made a transfer of \$6,000 but the transfer cost him only \$4500. The other \$1,500 came from the reduction in his income taxes. If our donor had to finance the entire transfer, to say individual 3, himself, he would have made a transfer such that

$$\frac{\partial U_1}{\partial X_1} = \frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3} \ ; \ dX_3 = -dX_1 \tag{3.99}$$

where we have assumed away the interaction effects of other individuals. When the government shares in his transfer he will be willing to transfer until

$$\frac{\partial U_1}{\partial X_1} = -\left(\frac{\partial U_1}{\partial X_3} + \frac{\partial U_1}{\partial U_3}\frac{\partial U_3}{\partial X_3}\right)\frac{dX_3}{dX_1} \tag{3.100}$$

where $dX_3 = - dX_1 - S$ and S is the amount paid by the government via reduced taxes to individual 1. The maximum share of individual 1's transfer the government will pay is one-fourth. This will occur when the total transfer is \$6,000.

If the average tax rate on $50,000 is fifty percent individual 2 would owe $25,000 in taxes if he made no deductions. After a $25,000 contribution he would owe taxes on $25,000. With an average tax rate of forty percent the individual would owe $10,000 in taxes. If he made a charitable transfer of $25,000 the cost to individual 2 would be $10,000. The government would finance the other $15,000 via its reduced tax receipts from individual 2. In this case the government would be willing to pay up to sixty percent of individual 2's transfer costs.

In general, the higher an individual's income the larger the government's share of any transfer he makes. The precise size of the government's share of a transfer will be determined by the individual's income, the average and marginal tax schedules, the maximum deduction allowed for charitable contributions and the amount actually transferred by the individual. Feldstein [16; 17] has determined that size of an individual's charitable contribution will increase as his income increases.

Another reason an individual might contribute to a charitable organization with a tax deductible status is that he may not like the way the government is using his tax dollars. When he makes a charitable contribution, not only is the government subsidizing his transfer but, perhaps more importantly, he is able, within limits, to direct how his money is to be used. These limits are fairly important. We showed earlier that when an individual makes a transfer to an intermediary he is effectively telling the intermediary to do what he wants with the money or goods. When he makes only a small transfer to the intermediary he will probably have little say in how the intermediary allocates his transfer. If he makes a large transfer, he may have much to say. The board members of charitable organizations are more likely to be individuals who have made large donations than individuals who have made small donations [10, p. 12].

In summary, an individual's desire to make a transfer is based on the marginal costs and benefits that transfer yields. The individual will make a transfer when the marginal benefits are greater than the marginal costs. If more than one individual desires to make a transfer to some particular recipient, they will all join in making the transfer unless some individuals receive more utility from letting the other individuals make the entire transfer. The individuals who do not contribute to the joint transfer are called free riders. Occasionally an individual cannot make a direct transfer to individuals he desires to help because he is either ignorant of their identities or ignorant of how they might best be helped. In these cases he might make a transfer to an intermediary who is not shackled by such information constraints. Once he makes the transfer to the intermediary the transferor usually gives up effective control of his transfer. Finally, the government can influence the individual's incentive to make transfers by altering its tax policy toward charitable contributions.

CHAPTER IV. SUMMARY AND DIRECTIONS FOR FURTHER RESEARCH

Our purpose in writing this dissertation was to show that eleemosynary or charitable behavior is an economically rational and justifiable action undertaken by individuals in order to improve their own well-being. We desired to show what conditions would cause an individual to make voluntary transfers and how these conditions would be affected under varying assumptions concerning potential recipients and other potential donors. In the first chapter we listed twelve reasons why one individual might seem to voluntarily transfer resources to another individual. From this list we selected the interdependent utility function as the most promising vehicle for economic analysis.

In Chapter II we reviewed the commonly accepted theory of the consumer. In this theory, the consumer is assumed to have a strictly private utility function so that his utility is a function of the goods and services which he himself consumes. The consumption and welfare levels of other individuals do not enter his utility function. Each of these consumers is economically selfish. An economically selfish individual will never voluntarily make a transfer to another individual since the transfer will lower his holdings of goods. If the individual has fewer goods to consume, his utility level will fall.

When the assumption that all consumers have strictly private utility functions is relaxed, the possibility of voluntary transfers is introduced. With an interdependent utility function, an individual receives utility from the consumption and welfare levels of other individuals as well as from his own consumption. If an individual has benevolent feelings for some individuals who enter his utility function, then increases in their consumption and welfare levels will increase his own utility level. His utility level will fall with increases in the consumption and welfare levels of the individuals he dislikes. The individual will make a transfer whenever the marginal benefits of the transfer are greater than the marginal costs. Transfers are generally motivated by the transferor's benevolent feelings for the recipient. Occasionally an individual may make a transfer whose primary purpose is not to increase the utility of the recipient but rather to reduce the utility level of some third party who dislikes the recipient and who is disliked by the transferor.

The fact that an individual is willing to make voluntary transfers indicates that his indifference surfaces are not strictly convex to his origin. If his benevolence is strong enough, his indifference surfaces will be hyper-ellipsoids. (We assumed these indifference surfaces were circles in the two good case.) When this occurs, the individual's consumption set will contain a bliss point. The existence of a bliss point indicates that there is some distribution of goods which will maximize this individual's utility. The location of an individual's bliss point will be dependent upon the supplies of all goods available for consumption and his feelings for each of the individuals who enter his utility function.

If there is a bliss point for an individual, then there will be a region of charity and several regions of possible charity for that individual. If a distribution of goods places the individual in his region of charity, then that individual will be willing to transfer some amount of each good he possesses. If the distribution places him in a region of possible charity, he will be willing to transfer portions of some, but not all, of the goods he possesses.

The transfers an individual makes will generally not lead to the Pareto efficient locus unless the donor is originally in his region of charity. Post-transfer exchange will usually be necessary to reach the Pareto efficient locus.

When there are more than two individuals in the society, the post-transfer exchanges may lead to situations where further transfers are desired by some individuals. When some of the externalities caused by interdependencies are malevolent, the Pareto efficient region for society will expand. A transfer between two individuals will be a Pareto better move for those two individuals, but it will not be a Pareto better move for society if some third individual is made worse off by the transfer. The Pareto efficient region can also expand if two individuals have very strong benevolent feelings for each other. Neither will accept a transfer from the other since his utility will only be increased when the other accepts a transfer from him. The expansion of society's Pareto efficient region means the Pareto criterion will be a less powerful tool in determining which transfers or exchanges are in society's best interest.

When more than one individual enters benevolently into an individual's utility function, that individual will base the order of his transfers on the size of the externality he experiences from each of the other individuals. He will make his first transfer to the individual who imposes the largest externality. When the externalities imposed by two or more individuals have the same highest value, he will divide his transfer among these individuals. The transferor will continue to make transfers until the marginal benefits of the transfer are equal to the marginal costs.

The transferor need not make transfers to the poorest individuals in his society. These individuals do not necessarily have to enter his utility function. Even if they do enter his utility function, we must remember that the decision to transfer is based in part on the marginal benefits the donor receives from making the transfer. Wealth and income levels will be only two of the many factors which determine these marginal benefits. It should not surprise us if an individual makes a transfer to a reasonably wealthy individual, perhaps to one even wealthier than himself, while he ignores the plight of the impoverished. The individual is simply making transfers which maximize his own welfare.

When two or more individuals want to make a transfer to the same recipient, then there may be an incentive for some of these potential transferors to become free riders. Since we have assumed that all individuals are utility maximizers, an individual will only become a free rider if free riding gives him a greater level of utility than contributing to a joint transfer. Generally, the smaller an individual's share in a joint transfer scheme the greater the incentive for him to contribute to the transfer. Sometimes a donor will refuse to contribute to a joint transfer, not because he does not have benevolent feelings for the recipient, but because he feels that the size of the transfer favored by the other transferors is too large. When two or more transferors decide to jointly administer a transfer to a single (class of) recipient(s), they are, in effect, forming a charitable organization. It is the transferors' concern for the welfare of a common recipient that makes the formation of charitable organizations possible.

When other individuals with interdependent utility functions enter an individual's utility function, then the feelings of these other individuals will affect the desire of the individual to make or receive transfers. For example, the individual might make a transfer he would not otherwise have made if he knows that making this transfer will please some individual other than the recipient. The feelings of the other individuals are a form of social pressure. However, this type of social pressure does not threaten the individual with an externally imposed loss of utility if he does not fulfill the desires of the other individuals.

When two individuals enter each other's utility functions, they may desire to make transfers to each other. As long as the transfer bundles contain no goods in common, the transfers will be made given that each is willing to accept the other's transfer. When there are goods in common, the noncommon goods will be transferred. The common goods will only be transferred if one transferor is very insistent, and then only in conjunction with a transfer desired by the other donor.

We have given an economic rationale for the general eleemosynary behavior of individuals. There are many specific areas of eleemosynary behavior which can be studied in depth using the interdependent utility function. One of these specific areas concerns intra-family transfers. Standard expositions of consumer theory assume that it applies equally to a single individual and to a single family unit. These expositions internalize the intra-family externalities and assume away any problems concerning the intra-family distribution of goods. The interdependent utility function can be used to help explain family behavior, especially the behavior concerning the distribution of goods within the family.

Another potential use of the interdependent utility function is in the area of bequests. It can help explain why an individual would take out an insurance policy on his life and why he would leave his possessions to some relatives and friends but not to others.

A third use for the interdependent utility function is in analyzing redistribution through the public sector. The governmental welfare function can be viewed as a function of the utility functions of all individuals under its jurisdiction. The utility functions of the top officials in a government agency will probably carry the most weight in that agency's welfare function, however.

Finally the interdependent utility function can have an impact on general equilibrium theory. We have utilized the interdependent utility function to study the transfers made by one individual to another. We did note that when an individual is in a region of possible charity he will prefer to engage in exchange with the desired recipient by trading those goods having relevant externalities for other goods. He will be willing to exchange the goods at terms of trade more favorable than the recipient can otherwise find on the market. This means that all individuals need not face the same set of relative prices. General equilibrium theory should be extended or reformulated to accommodate this result.

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